**πgr-CLOSED SETS IN TOPOLOGICAL SPACES**

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**ABSTRACT**

The aim of this paper is to introduce a new class of sets called πgr-closed sets in topological spaces and to study their properties. Further, we define and study πgr-continuity, πgr-irresolute maps and πgr-T₁/₂-Space.

**INTRODUCTION**


\[ RO(X), πO(X), SO(X) \]

Definition 2.2

A subset A of topological space X is said to be

(i) \( a \) ω-closed [14] if \( cl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \in SO(X) \).

(ii) a generalized closed set [11] (g-closed set) if \( cl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \in O(X) \).

(iii) a regular generalized closed set [16] (briefly rg-closed set) if \( cl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \in RO(X) \).

(iv) a weakly generalized closed [14] (briefly wg-closed) if \( cl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \in O(X) \).

(v) a πg-closed set [10] if \( acl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \in πO(X) \).

(vi) a semi-pre-open set [1] if \( A \subseteq cl(int(cl(A))) \) and a semi-pre-open set if \( int(cl(int(A))) \subseteq A \).

(vii) a b-open set [2] if \( A \subseteq cl(int(A)) \cup int(cl(A)) \) and its complement is b-closed.

The family of all open sets [regular open, π-open, semi-open] sets of X will be denoted by O(X) resp.

\[ RO(X), πO(X), SO(X) \]

**Definition 2.1**

A subset A of a topological space X is said to be

(i) a pre-open [13] if \( A \subseteq int(cl(A)) \) and pre-closed if \( A \subseteq cl(int(A)) \).

(ii) a semi-open [1] if \( A \subseteq cl(int(A)) \) and semi-closed if \( cl(A) \subseteq A \).

(iii) a regular open [16] if \( A \subseteq cl(int(A)) \) and regular closed if \( A = cl(int(A)) \).

(iv) a α-open [12] if \( A \subseteq cl(int(cl(A))) \) and α-closed if \( cl(cl(A)) \subseteq A \).

(v) a π-open [24] if \( A \) is the finite union of regular open sets and the complement of π-open is π-closed set in X.

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**Key Words:** - πgr-closed sets, πgr-continuous, πgr-irresolute maps, πgr-T₁/₂ space.

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Definition 2.3
A map \( f: X \rightarrow Y \) is said to be
(1) a continuous function \[1\] if \( f^{-1}(V) \) is closed in \( Y \), for every closed set \( V \) in \( Y \).
(2) a regular continuous \[16\] if \( f^{-1}(V) \) is regular closed in \( X \), for every closed set \( V \) in \( Y \).
(3) a semi continuous \[1\] if \( f^{-1}(V) \) is semi closed in \( X \), for every closed set \( V \) in \( Y \).
(4) a \( \omega \)-continuous \[14\] if \( f^{-1}(V) \) is \( \omega \)-closed in \( X \), for every closed set \( V \) in \( Y \).
(5) a rg-continuous \[8\] if \( f^{-1}(V) \) is rg-closed in \( X \), for every closed set \( V \) in \( Y \).
(6) a \( \pi \)-continuous \[6\] if \( f^{-1}(V) \) is \( \pi \)-closed in \( X \), for every closed set \( V \) in \( Y \).
(7) a \( \pi g \)-continuous \[6\] if \( f^{-1}(V) \) is \( \pi g \)-closed in \( X \), for every closed set \( V \) in \( Y \).
(8) a g-continuous \[11,7\] if \( f^{-1}(V) \) is \( g \)-closed in \( X \), for every closed set \( V \) in \( Y \).
(9) a gpr-continuous \[8\] if \( f^{-1}(V) \) is gpr-closed in \( X \), for every closed set \( V \) in \( Y \).
(10) a wg-continuous \[14\] if \( f^{-1}(V) \) is wg-closed in \( X \), for every closed set \( V \) in \( Y \).
(11) a \( \pi g r a \)-continuous \[10\] if \( f^{-1}(V) \) is \( \pi g r a \)-closed in \( X \), for every closed set \( V \) in \( Y \).
(12) a \( \pi g g \)-continuous \[18\] if \( f^{-1}(V) \) is \( \pi g g \)-closed in \( X \), for every closed set \( V \) in \( Y \).
(13) a \( \pi g s \)-continuous \[3\] if \( f^{-1}(V) \) is \( \pi g s \)-closed in \( X \), for every closed set \( V \) in \( Y \).
(14) a \( \pi g b \)-continuous \[22\] if \( f^{-1}(V) \) is \( \pi g b \)-closed in \( X \), for every closed set \( V \) in \( Y \).
(15) a \( \pi g p \)-continuous \[9\] if \( f^{-1}(V) \) is \( \pi g p \)-closed in \( X \), for every closed set \( V \) in \( Y \).
(16) a rga-continuous \[23\] if \( f^{-1}(V) \) is rga-closed in \( X \), for every closed set \( V \) in \( Y \).
(17) a \( \pi r \)-continuous \[15\] if \( f^{-1}(V) \) is \( \pi r \)-closed in \( X \), for every closed set \( V \) in \( Y \).
(18) a wrg-continuous \[19\] if \( f^{-1}(V) \) is wrg-closed in \( X \), for every closed set \( V \) in \( Y \).

Definition 2.4
The closure of a set \( A \) is defined as the intersection of all regular closed sets containing the set and the interior of the set. The above are denoted by rcl(A) and r int(A).

Definition 2.5
A map \( f: X \rightarrow Y \) is said to be
(i) a irresolute function \[16\] if \( f^{-1}(V) \) is regular closed in \( Y \) for every regular closed set \( V \) in \( Y \).
(ii) a regular irresolute \[16\] if \( f^{-1}(V) \) is regular closed in \( Y \) for every regular closed set \( V \) in \( Y \).

3. \( \pi g r \)-Closed Sets In Topological Spaces.
Definition 3.1
A subset \( A \) of \( X \) is called \( \pi g r \)-closed set in \( X \) if \( rcl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \in \pi \mathcal{O}(X) \).
We denote the family of all \( \pi g r \)-closed (resp. \( \pi g r \)-open) sets in \( X \) by \( \pi G R C(X) \) (resp. \( \pi G R O(X) \)).

Theorem 3.2
1. Every regular closed set is \( \pi g r \)-closed set.
Proof: Follows from the definition.
Remark 3.3
The converse of the above results need not be true as seen in the following example.
Example 3.4
Let \( X = \{a, b, c\} \), \( \tau = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}\} \), The set \( A = \{c\} \) is \( \pi g r \)-closed, but not regular closed.

Theorem 3.4
Every \( g^*r \)-closed set is \( \pi g r \)-closed.
Proof: Let \( A \) be \( g^*r \)-closed set and \( A \subseteq U \), where \( U \) is \( \pi \)-open. Then \( rcl(A) \subseteq U \), where \( U \) is \( \pi \)-open. Hence \( A \) is \( \pi g r \)-closed.

Remark 3.5
The converse of the above need not be true as seen in the following example.
Example 3.6
Let \( X = \{a, b, c, d\} \), \( \tau = \{\varnothing, X, \{c\}, \{d\}, \{c, d\}, \{b, c\}, \{a, c, d\}, \{b, c, d\}\} \). Then \( A = \{a, d\} \) is \( \pi g r \)-closed but not \( g^*r \)-closed.

Theorem 3.7
Every \( \pi g r \)-closed set is rg-closed.
Every \( \pi g r \)-closed set is \( \pi g p \)-closed.
Every \( \pi g r \)-closed set is \( \pi g s \)-closed.
Every \( \pi g r \)-closed set is \( \pi g s p \)-closed.
Every \( \pi g r \)-closed set is \( \pi g b \)-closed.
Every \( \pi g r \)-closed set is \( \pi g b \)-closed.
Every \( \pi g r \)-closed set is \( \pi g b \)-closed.
Every \( \pi g r \)-closed set is \( \pi g b \)-closed.
Every \( \pi g r \)-closed set is \( \pi g b \)-closed.

Proof: Straight forward.

Remark 3.8
The converse of the need not be true as shown in the following examples.
Example 3.9
Let \( X = \{a, b, c, d\} \), \( \tau = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c, d\}\} \). Then \( A = \{a, b, c\} \) is \( \pi g r \)-closed sets.

\[ \begin{align*}
\pi g r \text{-closed sets} & = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c, d\}\} \\
\pi g p \text{-closed sets} & = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c, d\}\} \\
\pi g s \text{-closed sets} & = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c, d\}\} \\
\pi g s p \text{-closed sets} & = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c, d\}\} \\
\pi g b \text{-closed sets} & = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c, d\}\} \\
\pi g r \text{-closed sets} & = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c, d\}\}
\end{align*} \]
**Example 3.10**

Let $X = \{a,b,c,d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$. Then the set $A = \{b\}$ is $\pi$-closed and $\pi^*g$-closed in $X$ but not $\pi gr$-closed in $X$.

**Remark 3.11**

The concepts of semi-closed, $\pi gr$-closed sets are independent.

**Example 3.12**

Let $X = \{a,b,c,d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$. Then the set $A = \{b\}$ is $\pi gr$-closed but not $\pi\omega$-closed.

**Example 3.13**

The concepts of $wg$-closed set and $\pi gr$-closed set are independent and is shown in the following example.

**Example 3.14**

Let $X = \{a,b,c,d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$. Then the set $B = \{a\}$ is $\pi gr$-closed but not $wg$-closed.

**Remark 3.15**

The concepts of $rgw$-closed and $\pi gr$-closed are independent and is shown in the following example.

**Example 3.16**

Let $X = \{a,b,c,d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$.

Hence the collection of sets $\{\{b\}, \{a,b\}, \{a,d\}\}$ are $rgw$-closed but not $\pi gr$-closed.

Let $Y = \{a,b,c,d\}$, $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a,b\}\}$. Here the collection of sets $\{\{a\}, \{a,d\}\}$ are $\pi gr$-closed but not $rgw$-closed in $X$.

**Remark 3.19**

The concepts of $rg\alpha$-closed sets and $\pi gr$-closed sets are independent.

**Example 3.20**

In example 3.16, the set $A = \{a\}$ is $\pi gr$-closed and not $rg\alpha$-closed and the set $B = \{a\}$ is $rg\alpha$-closed but not $\pi gr$-closed.

**Remark 3.21**

The concepts of $\pi gr$-closed set and $P_{\tau}$-closed set are independent.

**Example 3.22**

In the above example 3.16, the set $A = \{a\}$ is $P_{\tau}$-closed but not $\pi gr$-closed and the set $B = \{a\}$ is $\pi gr$-closed but not $P_{\tau}$-closed.

**Remark 3.23**

The concepts of $\pi gr$-closed, $g$-closed, $\alpha$-closed, $\omega$-closed, pre-closed are independent to $\pi gr$-closed set.

**Example 3.24**

Let $X = \{a,b,c,d\}$, $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a,c\}, \{a,c,d\}\}$. Then the closed set $= \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{a,b,c,d\}\}$

$\pi gr$-closed set $= \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,b,c\}, \{a,b,d\}\}$

$\omega$-closed set $= \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,b,c\}, \{a,b,d\}\}$

**Remark 3.25**

The above discussions are shown in the following diagram.

**Remark 3.26**

The following is the diagrammatic representation of independent concepts of the sets with $\pi gr$-closed sets.

**Remark 3.27**

The Theorem of two $\pi gr$-closed sets is again a $\pi gr$-closed sets of $X$.

**Proof:**

Assume that $A$ and $B$ are $\pi gr$-closed sets in $X$. Let $U$ be $\pi$-open set in $X$ such that $A \cup B \subset U$. Then $A \subset U$ and $B \subset U$. Hence $A \cap B = \emptyset$. That is $A \cap B \subset \emptyset$. Hence $A \cap B$ is an $\pi gr$-closed set in $X$.

**Remark 3.28**

The intersection of two $\pi gr$-closed sets need not be $\pi gr$-closed. The fact given above is shown in the following example.

**Example 3.29**

Let $X = \{a,b,c,d\}$, $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a,c\}, \{a,c,d\}\}$. Let $A = \{a\}$ and $B = \{d\}$ are $\pi gr$-closed sets. But their intersection $\{a\}$ is not $\pi gr$-closed.

**Theorem 3.30**

If a subset $A$ of $X$ is both $\pi$-open and are $\pi gr$-closed, then it is closed.

**Proof:**

Assume that $A$ and $B$ are $\pi$-open set in $X$ such that $A \cup B \subset \emptyset$. Then $A \subset \emptyset$ and $B \subset \emptyset$. Hence $A \cap B = \emptyset$. That is $A \cap B \subset \emptyset$. Hence $A \cap B$ is an $\pi gr$-closed set in $X$.
Let $A$ be a subset of $X$ which is both are $\pi$-open and are $\pi gr$-closed. Then $rcrA\subset U$ whenever $A\subset U$ and $U$ is $\pi$-open. Also, $A\subset rclA$, which implies $rcrA=A$.

$\implies A$ is regular closed. Hence $A$ is closed.

**Theorem 3.31**

If $A$ is $\pi gr$-closed set and $A\subset B\subset rclA$, then $B$ is also $\pi gr$-closed set of $X$.

Proof:

Let $A$ be $\pi gr$-closed set in $X$ and $B\subset U$, where $U$ is $\pi$-open. Since $A\subset B\subset U$. Since $A$ is $\pi gr$-closed, $rclA\subset U$. Given $B\subset rclA$, then $rclB\subset rclA\subset U$.

$\implies rclB\subset U$ and hence $B$ is $\pi gr$-closed.

**Theorem 3.32**

If $A$ is $\pi gr$-closed set, then $rcl(A) - A$ does not contain any non-empty $\pi gr$-closed set.

Proof:

Let $F$ be a non-empty $\pi$-closed set such that $F\subset rcl(A)-A$.

$\implies F\subset A$. The above implies $A\subset F$. Since $A$ is $\pi gr$-closed, $X-F$ is $\pi$-open.

Since $rclA\subset F$, $F\subset rclA$. Thus, $F\subset rclA\cap(X-rclA)$.

$\implies F\subset \varnothing$, which is a contradiction.

Thus $F=\varnothing$, whence $rcl(A)-A$ does not contain any non-empty $\pi gr$-closed set.

**Corollary 3.33**

Let $A$ be $\pi gr$-closed set in $X$. Then $A$ is regular closed iff $rcl(A)-A$ is $\pi gr$-closed.

**Proof**:

Necessity: Let $A$ be regular closed. Then $rcl(A)=A$ and so $rcl(A)-A=\varnothing$, which is $\pi gr$-closed.

Sufficiency: Suppose $rcl(A)-A$ is $\pi$-closed. Then $rcl(A)-A=\varnothing$, since $A$ is $\pi gr$-closed.

(i.e.) $rcl(A)=A$. $\implies A$ is regular closed.

**4. $\pi gr$-Open Sets**

**Definition 4.1**

A set $A\subset X$ is called $\pi gr$-open set iff its complement is $\pi gr$-closed.

The collection of all $\pi gr$-open sets is denoted by $\pi gro(X)$.

**Remark 4.2**

For a subset $A$ of $X$, $rcr(X-A)=X-rint(A)$.

**Theorem 4.3**

Let $A\subset X$ is $\pi gr$-open iff $F\subset rint(A)$ whenever $A$ is $\pi$-closed and $F\subset A$.

**Proof**:

Necessity: Let $A$ be $\pi gr$-open. Let $F$ be $\pi$-closed set and $F\subset A$. Then $X-A\subset X-F$, where $X-F$ is $\pi$-open.Since $A$ is $\pi gr$-open, $X-A$ is $\pi gr$-closed. Then $rcr(X-A)<X-F$.

Since $rcr(X-A)=X-rint(A)$. The above implies $X-rint(A)<X-F$. Hence $F\subset rint(A)$.

Sufficiency: Suppose that $F$ is $\pi$-closed and $F\subset A$ implies $F\subset rint(A)$. Let $X-A\subset U$, where $U$ is $\pi$-open. Then $X-U\subset A$. By hypothesis, $X-U\subset rint(A)$. $\implies X-rint(A)\subset U$. Since $rcr(X-A)=X-rint(A)$. The above implies $rcr(X-A)\subset U$, whenever $X-A$ is $\pi$-open. Hence $X-A$ is $\pi gr$-closed and hence $A$ is $\pi gr$-open.

**Theorem 4.4**

If $rint(A)\subset B\subset A$ and $A$ is $\pi gr$-open, then $B$ is $\pi gr$-open.

Proof:

Given $rint(A)\subset B\subset A$. The $X-A\subset X-B\subset rcl(X-A)$.

Since $A$ is $\pi gr$-open, $X-A$ is $\pi gr$-closed. Then $X-B$ is also $\pi gr$-closed. Hence $B$ is $\pi gr$-open.
(i) Let A be a regular open. Then X-A is regular closed and so πgr-closed.

⇒A is πgr-open. Hence RO(X) ⊆ πGRO(X)

(ii) Necessity: Let X be πgr-T1/2-space. Let A ∈ πGRO(X).

Then X-A is πgr-closed. Since the space X is πgr-T1/2-space, X-A is regular closed. The above implies A is regular open in X.

Hence RO(X)=πGRO(X)

Sufficiency: Let RO(X)=πGRO(X).

Let A be πgr-closed. Then X-A is πgr-open and X-A ∈ RO(X).

Hence A is regular closed and hence a πgr-T1/2-space.

6. πgr-Continuous and πgr-irresolute functions

Definition 6.1
A function f: (X,τ) → (Y, σ) is called πgr-continuous if every f⁻¹(V) is πgr-closed in X for every closed set V of Y.

A function f: (X,τ) → (Y, σ) is called πgr-irresolute if every f⁻¹(V) is πgr-closed in X for every πgr-closed set V of Y.

Example 6.2
Let X={(a,b,c,d), τ = {φ, X, {a}, {b}, {c}, {d}, {a,b,c,d}, {b,c,d}, {c,d}}}. Then πgr-closed sets in Y are 

{φ, Y, {b,c,d}, {a,b,c,d}}

Let Y={(a,b,c,d), σ = {φ, Y, {a,c,d}}}. Let f: (X,τ) → (Y, σ) by f(a)=a, f(b)=b, f(c)=c, f(d)=d. Here the inverse image of the closed set in Y is πgr-closed in X. Hence the function f is πgr-continuous.

Remark 6.3
Every regular continuous is πgr-continuous but not conversely.

Proof: straight forward

Example 6.4
Let X = {a,b,c,d}, τ = {φ, X, {a}, {b}, {c}, {d}, {a,b}, {b,c}, {c,d}, {a,b,c,d}, {a,c,d}, {b,c,d}, {a,b,c,d}}.

Let Y = {a,b,c,d}, σ = {φ, Y, {a,c,d}}, σ c = {φ, Y, {a,c,d}}.

Define f: (X,τ) → (Y, σ) by f(a)=a, f(b)=b, f(c)=c, f(d)=d. Here the inverse image of the closed set (a,c,d) in Y is πgr-closed in X but not regular closed in X. Hence πgr-continuous function need not be regular continuous in X.

Remark 6.5
Every continuous, g-continuous, ω-continuous, semi-continuous, pre-continuous, rw-continuous, wg-continuous, Pr-continuous and rgu-continuous are independent of πgr-continuous. The facts given above are shown in the following examples.

Example 6.6
Let X = {a,b,c,d}, τ = {φ, X, {a}, {b}, {c}, {d}, {a,b,c,d}}.

Let Y = {a,b,c,d}, σ = {φ, Y, {a}, {b}, {c}, {d}, {a,b,c,d}, {a,c,d}, {b,c,d}}.

Define f: (X,τ) → (Y, σ) by f(a)=a, f(b)=b, f(c)=c, f(d)=d. Here the inverse image of the closed set (b) in Y is not πgr-closed in X but πg-closed, πg-closed, πgu-closed, πgb-closed, πcg-closed, πgw-closed, πrw-closed in X. Hence πg-continuous, πgw-continuous, πrgc-continuous, πrgc-continuous, πrgc-continuous functions need not be πgr-continuous.

Remark 6.13
The above discussions are summarized in the following diagrammatic representation.

![Diagram showing the relationships between different types of continuity](image)

Remark 6.14
The composition of two πgr-continuous functions need not be πgr-continuous in the following example.
Example 6.15
Let $X = \{a, b, c, d\}$, $Y = Z$, $\tau = \{\emptyset, X, \{a, b, c, d\}, \{d\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}\}$, $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{c\}, \{a, b\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions. Then

(i) $(gof)^{-1}$ is $\pi gr$-continuous if $f$ is $\pi gr$-continuous and $g$ is $\pi gr$-continuous.

(ii) $(gof)^{-1}$ is $\pi gr$-irresolute if $f$ is $\pi gr$-irresolute and $g$ is $\pi gr$-irresolute.

(iii) $(gof)^{-1}$ is regular $\pi gr$-continuous if $f$ is regular $\pi gr$-continuous and $g$ is regular $\pi gr$-continuous.

Proof:
(i) Let $V$ be regular closed in $Z$. Then $g^{-1}(V)$ is regular closed in $Y$. Since $g$ is regular continuous, $(gof)^{-1}(V)$ is $\pi gr$-closed in $X$. Hence $(gof)^{-1}$ is $\pi gr$-continuous.

(ii) Let $V$ be closed in $Z$. Since $g$ is $\pi gr$-continuous, $g^{-1}(V)$ is $\pi gr$-closed in $Y$. As $f$ is $\pi gr$-irresolute, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is $\pi gr$-closed in $X$. Hence $(gof)^{-1}$ is $\pi gr$-irresolute.

(iii) Let $V$ be a closed set in $Z$. Since $g$ is $\pi gr$-continuous, $g^{-1}(V)$ is $\pi gr$-closed in $Y$. As $Y$ is a $\pi gr T_\frac{1}{2}$-space, $g^{-1}(V)$ is regular closed in $Y$, $g^{-1}(V)$ is closed in $Y$. Hence $(gof)^{-1}$ is regular closed in $X$ and hence $(gof)^{-1}$ is regular continuous.

Theorem 6.16

Every $\pi gr$-irresolute function is $\pi gr$-continuous but not conversely.

Proof:
Follows from the definitions.

Example 6.17

Let $X = \{a, b, c\}$, $Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b, c\}\}$, $\sigma = \{\emptyset, Y, \{a, b\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions. Here the inverse image of the closed sets in $Y$ are $\pi gr$-closed in $X$ but the inverse image of the $\pi gr$-closed sets in $Y$ are not $\pi gr$-closed in $X$. Hence their composition $(gof)^{-1}$ is not $\pi gr$-continuous.

Theorem 6.18

The composition of two $\pi gr$-irresolute functions is $\pi gr$-irresolute.

Proof:
Straightforward.

Theorem 6.19

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be any two functions. Then

(i) $(gof)^{-1}$ is $\pi gr$-continuous if $f$ is regular continuous and $g$ is $\pi gr$-continuous.

(ii) $(gof)^{-1}$ is $\pi gr$-irresolute if $f$ is $\pi gr$-irresolute and $g$ is $\pi gr$-irresolute.

(iii) $(gof)^{-1}$ is regular $\pi gr$-continuous if $f$ is $\pi gr$-continuous and $g$ is regular $\pi gr$-continuous.

REFERENCES