INTRODUCTION

After the introduction of fuzzy sets by Zadeh [13] in 1965 and fuzzy topology by Chang [3] in 1967, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. After the introduction of intuitionistic fuzzy sets by Atanassov, [1] various authors have turned their attentions to this concept and it becomes the primary aim of many mathematicians to examine and explore how far the basic concepts and Theorems remain true.

In the last 25 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [5] introduced the concept of intuitionistic fuzzy topological spaces. Recently many fuzzy topological concepts such as semi closed, α closed, semi preclosed have been generalized for intuitionistic fuzzy topological spaces. In the present paper we introduce the concepts of intuitionistic fuzzy π -generalized semi pre closed set and intuitionistic fuzzy π -generalized semi pre open sets and study some of their properties.

In this paper we discuss the basic Definitions and results used in this paper. In section 2 we discuss about intuitionistic fuzzy π -generalized semi pre closed sets and in section 4 we discuss intuitionistic fuzzy π -generalized semi pre open sets.

2. PRELIMINARIES

Let (X, τ) be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form A = {< x, μA(x), νA(x) > : x ∈ X }, where the functions μA : X → [0,1] and νA : X → [0,1] denotes the degree of membership μA(x) and the degree of non membership νA(x) of each element x ∈ X to the set A respectively and 0 ≤ μA(x)+ νA(x) ≤ 1 for each x ∈ X. The intuitionistic fuzzy sets in (X, τ) are respectively called empty and whole intuitionistic fuzzy set on X. An intuitionistic fuzzy set A = {< x, μA(x), νA(x) > : x ∈ X } is called a subset of an intuitionistic fuzzy set B = {< x, μB(x), νB(x) > : x ∈ X } (for short A ⊆ B) if μA(x) ≤ μB(x) and νA(x) ≥ νB(x) for each x ∈ X.

The complement of an intuitionistic fuzzy set A = {< x, μA(x), νA(x) > : x ∈ X } is the union of all intuitionistic fuzzy closed sets in X. The intersection (resp. union) of any arbitrary family of intuitionistic fuzzy sets is given by A ∩ B = {< x, μA(x) ∨ μB(x), νA(x) ∧ νB(x) > : x ∈ X} and A ∪ B = {< x, μA(x) ∧ μB(x), νA(x) ∨ νB(x) > : x ∈ X }.

A family τ of intuitionistic fuzzy sets on a non empty set X is called an intuitionistic fuzzy topology on X if τ is closed under arbitrary union and finite intersection. The ordered pair (X, τ) is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in τ is called an intuitionistic fuzzy open set. The complement of an intuitionistic fuzzy open set in X is known as intuitionistic fuzzy closed set.

The intersection of all intuitionistic fuzzy closed sets which contains A is called the closure of A. It denoted by cl(A). The union of all intuitionistic fuzzy open subsets of A is called the interior of A. It is denoted by int (A) [5], as follows:

\[ \text{int}(A) = A \cap \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \} = \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \} \]

For the sake of simplicity, we use the notation A = < x, μA, νA > instead of A = {< x, μA(x), νA(x) > : x ∈ X } and A = < x, μA, νA > instead of A = < x, (μA, μB), (νA, νB) > instead of A = < x, (μA, μB, νA, νB) >.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement AC of an IFS in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X. Note that for any IFS A in (X, τ), cl(AC) = (int(A))c and int(AC) = (cl(A))c.

Definition 2.1: [7] A subset of A of a space (X, τ) is called:
(i) regular open if A = int (cl(A)).
(ii) π open if A is the union of regular open sets.

Definition 2.2: [4] An IFS A = < x, μA, νA > in an IFTS (X, τ) is said to be an intuitionistic fuzzy semi closed set (IFSCS in short) if int(cl(A)) ⊆ A.

Definition 2.3: [4] An IFS A = < x, μA, νA > in an IFTS (X, τ) is said to be an intuitionistic fuzzy semi open set (IFSOS in short) if A ⊆ int(cl(A)).

Definition 2.4: [4] An IFS A of an IFTS (X, τ) is an (i) intuitionistic fuzzy pre closed set (IFPCS in short) if cl(int(cl(A)) ⊆ A, (ii) intuitionistic fuzzy pre open set (IFPOS in short) if A ⊆ int(cl(A)).

Definition 2.5: [4] An IFS A of an IFTS (X, τ) is an (i) intuitionistic fuzzy α-open set (IFαOS in short) if A ⊆ int(cl(A)), (ii) intuitionistic fuzzy α-closed set (IFαCS in short) if cl(int(cl(A))) ⊆ A.

Definition 2.6: [2] An IFS A of an IFTS (X, τ) is an (i) intuitionistic fuzzy γ-open set (IFγOS in short) if A ⊆ int(cl(A)) ∪ cl(int(A)), (ii) intuitionistic fuzzy γ-closed set (IFγCS in short) if cl(int(cl(A))) ∩ int(cl(A)) ⊆ A.

Definition 2.7: [8] An IFS A of an IFTS (X, τ) is an (i) intuitionistic fuzzy regular open set (IFROS in short) if A = int(cl(A)), (ii) intuitionistic fuzzy regular closed set (IFRCS in short) if A = cl(int(cl(A))).

Definition 2.8: [12] An IFS A of an IFTS(X, τ) is called an intuitionistic fuzzy w-closed (IFWCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFWSO in X. An IFS A of an IFTS(X, τ) is called an intuitionistic fuzzy w-open (IFWOS in short) if AC is IFWCS.

Definition 2.9: [8] An IFS A of an IFTS (X, τ) is an intuitionistic fuzzy generalized closed set (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFGSO in X.

Definition 2.10: [6] Let an IFS A of an IFTS (X, τ) be a semi closure of A (scl(A) in short) is defined as scl(A) = ∩{ K | K is an interior and the semi-pre closure of A are defined as

spcl(A) = ∪ { K | K is an IFSPCS in X and A ⊆ K}.

Definition 2.11: [6] Let A be an IFS of an IFTS (X, τ). Then the semi interior of A (sint(A) in short) is defined as sint(A) = ∪ { K | K is an IFSPCS in X and K ⊆ A}.

Definition 2.12: [9] An IFS A of an IFTS (X, τ) is an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IFGSO in X.

Definition 2.13: [4] An IFS A of an IFTS (X, τ) is an intuitionistic fuzzy semi pre open set (IFSPOS in short) if there exists an IFSPSO B such that B ⊆ A ⊆ B.

Definition 2.14: [4] An IFS A of an IFTS (X, τ) is an intuitionistic fuzzy semi pre closed set (IFSPCS in short) if there exists an IFSPCS B such that int(B) ⊆ A ⊆ B.

Note that an IFS A is an IFSPCS if and only if int(cl(int(A))) ⊆ A.

Definition 2.15: [15] Let A be an IFS in an IFTS (X, τ). Then the semi-pre interior and the semi-pre closure of A are defined as

spint(A) = ∪ { G | G is an IFSPSO in X and G ⊆ A}

spcl(A) = ∩ { K | K is an IFSPCS in X and A ⊆ K}.

Note that for any IFS A in (X, τ),

spcl(AC) = { spint(A)}c and spint(AC) = {spcl(A)}c [15].

Result 2.16: Let A be an IFS in (X, τ), then

(i) spcl(A) = A ∪ int(cl(int(cl(A)))).
(ii) spint(A) = A ∩ cl(int(cl(A))).

Remark: 2.17: [16] Every IFOS is IFOSO in (X, τ).

Result: 2.18: [10] Union of two IFROS is IFOS in (X, τ).

Remark 2.19: [10] Every IFπOS is IFOS in (X, τ).

3. INTUITIONISTIC FUZZY π -GENERALIZED SEMI PRE CLOSED SETS:

In this section we introduce Intuitionistic fuzzy π generalized semi-pre closed sets and study some of their properties.

Definition 3.1: An IFS A is said to be an intuitionistic fuzzy π generalized semi-pre closed sets

(IFπGSPCS in short ) if spcl(A) ⊆ U whenever A ⊆ U and U is IFπOS in X.

Example 3.2: Let X = {x1, x2} and let τ = {0~, G, 1~} is an IFT on X, where G = {x < x, (0.3, 0.4), (0.7, 0.5)} Then the IFS A = < x, (0.2, 0.3), (0.7, 0.7) > is an IFπGSPCS in X.

Theorem 3.3: Every IFCS is an IFπGSPCS but not conversely.

Proof: Let A be an IFCS in X and let A ⊆ U and U is an IFπOS in (X, τ). Since spcl(A) ⊆ cl(A) ⊆ cl(A) and A is an IFCS in X, spcl(A) ⊆ cl(A) ⊆ A ⊆ U. Therefore A is an IFπGSPCS in X.

Example 3.4: Let X = {x1, x2} and let τ = {0~, G, 1~} is an IFT on X, where G = {x < x, (0.2, 0.2), (0.3, 0.3), (0.7, 0.7)} Then the IFS A = < x, (0.3, 0.3), (0.7, 0.7) > is an IFπGSPCS in X.

Theorem 3.5: Every IFπOS is an IFπGSPCS but not conversely.

Proof: Let A be an IFπOS in X and let A ⊆ U and U is an IFπOS in (X, τ). By hypothesis, spcl(A) ⊆ cl(A) ⊆ cl(A) and A is an IFπOS in X, spcl(A) ⊆ cl(A) ⊆ A ⊆ U. Therefore A is an IFπGSPCS in X.

Example 3.6: Let X = {x1, x2} and let τ = {0~, G, 1~} where G = {x < x, (0.3, 0.4), (0.4, 0.6)} Then the IFS A = < x, (0.3, 0.7), (0.5, 0.7) > is an IFπGSPCS in X. Hence spcl(A) = A ⊆ U but not an IFπGSPCS in X, since int(cl(A)) = {x < (0.3, 0.4), (0.4, 0.6)} ⊆ A.

Theorem 3.7: Every IFOSC is an IFπGSPCS but not conversely.

Proof: Let A be an IFOSC in X and let A ⊆ U and U is an IFπOS in (X, τ). By hypothesis, cl(int(cl(A))) ⊆ A. Therefore int(cl(A)) ⊆ A. Also, Int(A)⊆ A, cl(A) ⊆ cl(A), hence int(cl(A)) ⊆ int(cl(A)) ⊆ A. This implies spcl(A) ⊆ A ⊆ U. Therefore A is an IFπGSPCS in X.

Example 3.8: Let X = {x1, x2} and let τ = {0~, G, 1~} where G = {x < x, (0.2, 0.2), (0.6, 0.7)} and G2 = {x < x, (0.4, 0.5), (0.5, 0.6)} and A = < x, (0.3, 0.2), (0.6, 0.5) >, then spcl(A) = G = U is an IFπGSPCS but not an IFπCS in X, since cl(int(cl(A))) = {x < (0.5, 0.5), (0.4, 0.5)} ⊆ A.

Theorem 3.9: Every IFπGSPCS is an IFπGSPCS but not conversely.

Proof: Let A be an IFπGSPCS in X and let A ⊆ U and U is an IFπOS in (X, τ). By hypothesis, cl(int(cl(A))) ⊆ A. Therefore int(cl(A)) ⊆ A. Also, Int(A)⊆ A, cl(A) ⊆ cl(A), hence int(cl(A)) ⊆ int(cl(A)) ⊆ A. This implies spcl(A) ⊆ A ⊆ U. Therefore A is an IFπGSPCS in X.

Example 3.10: Let X = {x1, x2} and G = {x < x, (0.3, 0.4), (0.7, 0.5)} and let τ = {0~, G, 1~} be an IFT on X. The IFS A = G, then spcl(A) = G ⊆ U is an IFπGSPCS but not an IFπCS in X, because cl(int(cl(A))) = {x < (0.7, 0.5), (0.3, 0.4)} ⊆ A.

Theorem 3.11: Every IFπGSPCS is an IFπGSPCS but not conversely.

Proof: Let A be an IFπGSPCS in X. By hypothesis, spcl(A) ⊆ A, whenever A ⊆ U and U is IFπOS. By (Remark 2.19) spcl(A)
Proof: Let A be an IFCS in X. By Definition 3.3, 3.12: Let X = \{ x1, x2 \} and let \tau = \{ {0}, G1, G2, 1{\overline{0}} \} be an IFT on X, where G1 = \{ x, (0.5,0.4), (0.3,0.3) \} >, G2 = \{ x, (0.4,0.3), (0.3,0.3) \} >. Then IFS A = \{ x, (0.3,0.4), (0.3,0.4) \} > is an IFGSCS but not an IFGCS in X since A \cup \text{Int}(\text{cl}(\text{Int}(A))) = 1 \sim \prec G1.

Theorem 3.13: Every IFCS is an IFGCS but not conversely.

Proof: Let A be an IFCS in X. By Definition 3.3. A \in \text{Int}(\text{cl}(\text{Int}(A))). This implies cl(A) = \text{cl}(\text{Int}(A)). Therefore A is an IFCS in X. By Theorem 3.3, 3.14: Let X = \{ x1, x2 \} and \tau = \{ {0}, G1, G2, 1{\overline{0}} \} be an IFT on X, where G1 = \{ x, (0.1,0.2), (0.5,0.6) \} > and let \tau = \{ {0}, G1, G2, 1{\overline{0}} \} be an IFT on X. The IFS A = G is an IFCS but not an IFGCS since \text{cl}(\text{Int}(A)) = \{ x, (0.5,0.6), (0.1,0.2) \} \sim \prec G.

Theorem 3.25: Every IFGCS is an IFGSCS but not conversely.

Proof: Let A be an IFGCS in X. By hypothesis, \text{spcl}(A) \subseteq U, whenever A \subseteq U and U is IFOS. By hypothesis and Remark 3.21 \text{spcl}(A) \subseteq \text{cl}(\text{Int}(A)) \subseteq \text{Int}(U) \subseteq U. This implies \text{spcl}(A) \subseteq U whenever A \subseteq U and U is IFOS. Therefore A is an IFGCS in X.

Example 3.26: Let X = \{ x1, x2 \} and let \tau = \{ {0}, G1, G2, 1{\overline{0}} \} be an IFT on X, where G1 = \{ x, (0.5,0.6), (0.3,0.3) \} >, G2 = \{ x, (0.4,0.3), (0.3,0.3) \} >. Then IFS A = G is an IFGSCS but not an IFGCS in X since A \cup \text{Int}(\text{cl}(\text{Int}(A))) = 1 \sim \prec G1.

Theorem 3.27: Every IFGSCS is an IFGCS but not conversely.

Proof: Let A be an IFGSCS in X. By hypothesis, \text{spcl}(A) \subseteq U, whenever A \subseteq U and U is IFOS. By hypothesis and Remark 3.21 \text{spcl}(A) \subseteq \text{cl}(\text{Int}(A)) \subseteq \text{Int}(U) \subseteq U. Therefore A is an IFGSCS in X.

Example 3.28: Let X = \{ x1, x2 \} and let \tau = \{ {0}, G1, G2, 1{\overline{0}} \} be an IFT on X, where G1 = \{ x, (0.1,0.3), (0.4,0.3) \} >, G2 = \{ x, (0.0,0.2), (0.2,0.3) \} >. Then IFS A = G is an IFGSCS but not an IFGCS in X since A \cup \text{Int}(\text{cl}(\text{Int}(A))) = 1 \sim \prec G1.
Hence A is an IFSPCS in X.

**Theorem 3.33:** Let (X, τ) be an IFTS. If an IFS A is both IFnOS and IFCS of X, then the following statements are equivalent:

(i) A is IFGCS in X
(ii) A is IFπGSPCS in X.

**Proof:** (i) ⇒ (ii): Let A be an IFGCS in X. By Theorem 3.17, A is IFFSPCS in X. (ii) ⇒ (i): Let A be an IFπFSPCS in X. Then spcl(A) ⊆ U whenever A ⊆ U and U is IFnOS in X, implies spcl(A) ⊆ cl(A) ⊆ U whenever A ⊆ U, Since A is both IFnOS and IFCS in X. Therefore A is IFπGCS in X.

### 4. INTUITIONISTIC FUZZY ρ-GENERALIZED SEMI PRE OPEN SETS:

In this section we introduce intuitionistic fuzzy ρ-generalized semi pre open sets and discuss some of its properties.

**Definition 4.1:** An intuitionistic fuzzy ρ-generalized semi pre open sets (IFπGSPos) in short (X, τ) if its complement AC is an IFπFSPCS in X. The family of all IFπGCS of an IFTS (X, τ) is denoted by IFπGSPCSX.

**Example 4.2:** Let X = {x₁, x₂} and let τ = {∅, G, 1~} is an IFT on X, where G = {< x₁, (0.3, 0.4), (0.2, 0.3) >}. Then τ = {∅, G, 1~} is an IFT in X. Then the IFS A = {< x₁, (0.6, 0.7), (0.2, 0.3) >} is an IFπGSPCS in X.

**Theorem 4.3:** For any IFTS (X, τ), we have the following:

Every IFOS, IFCS, IFπGSP, IFPOS, IFSPCS in X. But the converses are not true in general.

**Proof:** Straightforward.

**Example 4.4:** Let X = {x₁, x₂} and G = {< x₁, (0.2, 0.3), (0.3, 0.4) >}. Then τ = {∅, G, 1~} is an IFT in X. The IFS A = {< x₁, (0.3, 0.4), (0.2, 0.3) >} is an IFπGSP in (X, τ) but not an IFnOS in X.

**Example 4.5:** Let X = {x₁, x₂} and G = {< x₁, (0.3, 0.4), (0.4, 0.6) >}. Then τ = {∅, G, 1~} is an IFT in X. The IFS A = {< x₁, (0.5, 0.7), (0.3, 0.5) >} is an IFπGSP in X but not an IFnOS in X.

**Example 4.6:** Let X = {x₁, x₂} and let τ = {∅, G₁, G₂, 1~} is an IFT on X, where G₁ = {< x₁, (0.2, 0.3), (0.2, 0.3) >} and G₂ = {< x₁, (0.4, 0.5), (0.2, 0.5) >}. Then τ = {∅, G₁, G₂, 1~} is an IFπGSP in X but not an IFnOS in X.

**Example 4.7:** Let X = {x₁, x₂} and G = {< x₂, (0.2, 0.3), (0.6, 0.5) >}. Then τ = {∅, G, 1~} is an IFT in X. The IFS A = {< x₁, (0.6, 0.5), (0.2, 0.2) >} is an IFπGSP but not an IFGOS in X.

**Example 4.8:** Let X = {x₁, x₂} and G = {< x₁, (0.3, 0.4), (0.7, 0.5) >}. Then τ = {∅, G, 1~} is an IFT in X. The IFS A = {< x₁, (0.7, 0.5), (0.3, 0.4) >} is an IFπGSP but not an IFPOS.

**Example 4.9:** Let X = {x₁, x₂} and let τ = {∅, G₁, G₂, 1~} is an IFT on X, where G₁ = {< x₁, (0.4, 0.5), (0.3, 0.2) >} and G₂ = {< x₂, (0.2, 0.3), (0.3, 0.2) >}. Then τ = {∅, G₁, G₂, 1~} is an IFπGSP but not an IFPOS in X.

**Theorem 4.10:** Let (X, τ) be an IFTS. If A ∈ IFπGSP(X) then V ⊆ cl(int(cl(A))) whenever V ⊆ A and V is IFCS in X.

**Proof:** Let A ∈ IFπGSP(X). Then AC is an IFπGSPCS in X. Therefore spcl(Ac) ⊆ U whenever Ac ⊆ U and U is an IFnOS in X. This implies that int(cl(int(AC))) ⊆ U. Therefore, U ⊆ cl(int(cl(A))) whenever (UC) ⊆ A, and UC is IFCS in X. Replacing UC by V, we get V ⊆ cl(int(cl(A))) whenever V ⊆ A and V is IFCS in X.

**Theorem 4.11:** Let (X, τ) be an IFTS. Then, ∀ A ∈ IFπGSP(X) and ∀ B ⊆ IFCS(X), spint(A) ⊆ B ⊆ A implies B ∈ IFπGSP(X).

**Proof:** By hypothesis Ac ⊆ B ⊆ (spint(A)) c. Let Bc ⊆ U and U be an IFnOS. Since Ac ⊆ Bc, Ac ⊆ U. But AC is an IFπGSPCS, spcl(AC) ⊆ U. Also Bc ⊆ (spcl(Ac)) c = spcl(AC) (Result 2.15). Therefore spcl(Bc) ⊆ spcl(AC) ⊆ U. Hence Bc is an IFπGSPCS, which implies B is an IFπGSPS of X.

**Remark 4.12:** The union of any two IFπGSPSs need not be an IFπGSPS in general.

**Example 4.13:** Let X = {x₁, x₂} be an IFTS and let G₁ = {< x₁, (0.2, 0.4), (0.3, 0.4) >}, G₂ = {< x₂, (0.1, 0.3), (0.5, 0.4) >}. Therefore τ = {∅, G₁, G₂, 1~} is an IFT on X and the IFCSs A = {< x₁, (0.3, 0.4), (0.1, 0.3) >}, B = {< x₂, (0.5, 0.4), (0.0, 0.4) >} are IFπGSPSs but A ∪ B is not an IFπGSPS in X.

**Theorem 4.14:** An IFS A of an IFTS (X, τ) is an IFπGSPS if and only if A is an IFπCS and F ⊆ A.

**Proof:** Necessity: Suppose A is an IFπGSPS in X. Let F be an IFπCS and F ⊆ A. Then Fc is an IFnOS in X such that Ac ⊆ Fc. Since Ac is an IFπGSPS, spcl(Ac) ⊆ Fc. Hence (spint(A)) ⊆ Fc. Therefore F ⊆ spint(A).

Sufficiency: Let A be an IFCS of X and let F ⊆ spint(A) whenever F is an IFCS and F ⊆ A. Then Fc is an IFnOS in X. By hypothesis, (spint(A)) ⊆ Fc, which implies spcl(Ac) ⊆ Fc. Therefore Ac is an IFπGSPS of X. Hence A is an IFπGSPS of X.

**Theorem 4.15:** Let (X, τ) be an IFTS and A, B ⊆ X. If B is an IFπGSPS(X) and spint(B) ⊆ A then A ∪ B is an IFπGSPS(X).

**Proof:** Since B is an IFπGSPS(X) and spint(B) ⊆ A, spint(B) ∩ A = B ⊆ X, by Theorem 4.11, A ∪ B is an IFπGSPS(X).

### REFERENCES


15. Santhi R and Jayanti D, Intuitionistic fuzzy generalized semi pre closed sets (accepted).