SPECIAL KINDS OF GRAPHS - EDGE PRODUCT NUMBER

J. P. Thavamani*

Department of Mathematics, M.E.S. College, Nedumkandam, Idukki, Kerala, India.

ARTICLE INFO

Corresponding Author
J. P. Thavamani
Department of Mathematics, M.E.S. College, Nedumkandam, Idukki, Kerala, India.
thavamani-prem@yahoo.co.in

Key Words: Edge product function, edge product graph, edge product number of a graph, optimal edge function and optimal edge product function

ABSTRACT

A graph G(V, E) is said to be a sum graph if there exists a bijective labeling from the vertex set V to a set S of positive integers such that xy ∈ E if and only if f(x) + f(y) ∈ S. We introduce the edge as well as the product analogue of sum graphs. The edge product number of a graph G is the minimum number r so that G∪rK2 becomes an edge product graph. In this paper we define some different kinds of graphs and also investigate the edge product number of that graphs.

ARTICLE INFO

In this section we define some different kinds of graphs and find their edge product number.

Definition 3.1: Let d and p be non-negative integers such that k = (d + p) ≥ 2. Let M(d, p) denote a graph with vertex set V={u1, u2, ..., u2k, v1, v2, ..., vp} and the edge set E={e1, e2, ..., e(2k - 1), e2k, f1, f2, ..., fd} where e1 = u1u2, e2 = u2u3, ..., ep = vpu(2k – p + 1). Theorem 3.2: EPN(Md, p) = 1 for every odd positive integer d and non-negative integer p.

Proof: Consider a graph G(V, E) = (M(d, p))∪K2 for d and p are non-negative integers. For every odd positive integer d, there exists a positive integer s such that d = (2s - 1). Take 2s = 2k + 1. Let V = {u1, u2, ..., u2k, v1, v2, ..., vp}∪{w1, w2} be the vertex set and set E = {e1, e2, ..., ep = vpu(2k – p + 1), f1, f2, ..., fd} be the edge set of G. Let the elements of P = {2s, 2s + 2, ..., 2s + E(2k+1)} - {1}. The mapping f : E → P is an edge function of G, and F is the corresponding optimal edge function which make G∪rK2 an edge product graph are respectively called an optimal edge function and an optimal graph product function of G.
The edge product function $F$ satisfies $F(v) = 2(2^{k} + 2 + 2^{k+1}i)$ for every vertex $v$ in $V$. Also $F(v) = f(w_1w_2)$. Therefore $F(v) = F(w_1) = F(w_2)$.

**Example 3.9**: Let $G(V,E)$ be a 3-regular graph of order 10 and size 15. Then the graph $(G∪K_2)$ is the edge product graph and figure 4 shows that $EPN(G) = 2$.

The mapping $f : E → P$ is a bijection. The edge product function $F$ satisfies $F(v) = 2(2^{k} + 2 + 2^{k+1}i)$ for every vertex $v$ in $V$. Also $F(v) = f(w_1w_2)$. Therefore $F(v) = F(w_1) = F(w_2)$.

**Example 3.3**: If $M_{(1,3)}$ is a graph and $M_{(1,3)}∪K_2$ is the edge product graph. Then $EPN(M_{(1,3)}) = 1$ is given in figure 1.

**Figure 1**: The mapping $f : E → P$ is a bijection. The edge product function $F$ satisfies $F(v) = 2^{(2^{k} + 2 + 2^{k+1}i)}$ for every vertex $v$ in $V$. Also $F(v) = f(w_1w_2)$. Therefore $F(v) = F(w_1) = F(w_2)$.

**Example 3.3**: If $M_{(1,3)}$ is a graph and $M_{(1,3)}∪K_2$ is the edge product graph. Then $EPN(M_{(1,3)}) = 1$ is given in figure 1.

**Figure 2**: The graph is illustrated in figure 3.

**Theorem 3.5**: If $G$ be a 3-regular graph of order 10 and size 15 then the edge product number of $G$ is 1, i.e., $EPN(G) = 1$.

**Proof**: Let $G(V,E)$ be a 3-regular graph of order 10 and size 15 with $V = \{v_0, v_1, v_2, \ldots, v_{15}\}$ be the vertex set and the edge set be $E = \{v_0v_1, v_1v_2, v_2v_3, \ldots, v_{14}v_5\}$. The graph is illustrated in figure 3.

Consider $EPN(G) = 1$. Let $f : E → P$ be an optimal edge function of the graph $G$, where $P = \{a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2, e_1, e_2, f_1, f_2, \ldots\}$ and $F$ be the corresponding optimal edge product function of $f$. The edge product function $F$ of $e_{i1}$ is defined by $F(e_{i1}) = f(v_1v_0) × f(v_2v_0) × f(v_3v_0) = a_1 × x × c_1$.

For $k$ is odd: $g(e_{i1}) = 2^{2k} - 2^{1} + (i / 2)$ for $i = 1, 3, \ldots, (k+1), (2k)$ and $g(e_{i1}) = 2^{2k} - 2^{1} + (i / 2)$ for $i = 1, 3, \ldots, (k+1), (2k)$ and $2k$.

For $k$ is even: $g(e_{i1}) = 2^{2k} + 2^{1} + (i / 2)$ for $i = 1, 3, \ldots, (k+1), (2k)$ and $g(e_{i1}) = 2^{2k} + 2^{1} + (i / 2)$ for $i = 1, 3, \ldots, (k+1), (2k)$ and $2k$.

Let $S_k$ denote a graph with vertex set $V(C_{2k})$ and the edge set $E(C_{2k})$.
$v_{i1}, v_{i2}$ are vertices of $C_{2k}$ such that $f(u_{i1}) = 2^{(3k - 3 + i)}$ and $f(u_{i2}) = 2^{(5k - 1 - i)}$.

**Theorem 3.6:** If $S_k$ is a graph then $EPN(S_k) = 1$ for every odd positive integer $k \geq 2$ there exists a edge product graph of order $3k$ and size $4k$.

**Proof:** Consider a graph $G(V, E) = (S_k \cup K_2)$. The vertex set of $G$ is $V = V(S_k) \cup \{w_1, w_2\}$ and the edge set of $G$ is $E = E(S_k) \cup \{w_1w_2\}$ where $v_{i1}, v_{i2}$ are vertices of $C_{2k}$. A mapping $f : E(S_k) \rightarrow \mathbb{P}$ where $\mathbb{P} = \{2^{(k - 1)}, \ldots, 2^{(5k - 2)}\}$ is an optimal edge function of the graph $G$ and $F$ be its corresponding optimal edge product function of $f$. The edge function $f : E(S_k) \rightarrow \mathbb{P}$ is defined by $f(e) = g(e)$ for $e \in E(C_{2k})$, $f(e) = 2^{(5k - 1 - i)}$ for $e = u_{i1}$ and $f(e) = 2^{(3k - 3 + i)}$ for $e = u_{i2}$. Clearly the mapping $f$ is a bijection. The edge product function $F$ satisfies $F(v) = 2^{(8k - 4)}$ for every $v \in V(S_k)$. Also $F(v) = f(w_1w_2)$. That is, $F(v) = F(w_1) = F(w_2)$. Hence $EPN(S_k) \geq 1$ for every odd positive integer $k \geq 2$.

Therefore we get the desired result that $EPN(S_k) = 1$ for $k \geq 2$.

**Example 3.7:** The graph $S_4 \cup K_2$ is the edge product graph and figure 5 shows that $EPN(S_4) = 1$.

**REFERENCES**


Figure 5