**g^p-LOCALLY CLOSED SETS AND g^p-LOCALLY CLOSED FUNCTIONS**

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**ABSTRACT**

In this paper, we introduce a new class of sets g^p-Locally closed sets, g^p-Locally closed functions and their properties.

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**INTRODUCTION**

N.Levine [6] introduced the class of g-closed sets. M.K.R.S.Veerakumar introduced several generalized closed sets namely, g^+-closed sets, g^+-locally closed sets and g^+-lc functions. The authors [9] have already introduced g^p-closed sets and their properties. In this paper, we introduce g^p-Locally closed sets, g^p-Locally closed functions and their properties.

**2.PRELIMINARIES**

Throughout this paper, (X,τ) (or X) represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. A subset A of a space (X,τ), cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively. We recall the following definitions which are useful in the sequel.

**Definition2.1:** A subset A of a space (X,τ) is called a (i) **generalized closed** (briefly g-closed) set [6] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ); the complement of a g-closed set is called a g-open [6] set.

(ii) **regular open** [5] set if A=cl(int(A)) and regular closed [5] set if cl(int(A))=A.

(iii) **regular generalized closed** (briefly rg-closed) set [8] if cl(A) ⊆ U whenever A ⊆ U and U is regular open in (X,τ); the complement of a rg-closed set is called a rg-open [8] set.

(iv) **g-generalized closed** (briefly gc-closed) set [7] if acl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ); the complement of an gc-closed set is called an gc-open [7] set.

(v) **g^+-closed** set [12] if cl(A) ⊆ U whenever A ⊆ U and U is g^+-open in (X,τ); the complement of a g^+-closed set is called a g^+-open [12] set.

(vi) **g^p-closed** set [9] if pcl(A) ⊆ U whenever A ⊆ U and U is gc-open in (X,τ); the complement of a g^p-closed set is called a g^p-open [9] set.

(vii) **g^*-closed** set [14] if cl(A) ⊆ U whenever A ⊆ U and U is semi open in (X,τ); the complement of a g^*-closed set is called a g^*-open [14] set.

(viii) **g^+**-closed set [11] if cl(A) ⊆ U whenever A ⊆ U and U is g-open in (X,τ); the complement of a g^+-closed set is called a g^+-open [11] set.

**Definition2.2:** A subset S of a space (X,τ) is called a (i) **regular generalized locally closed** (briefly rglc) set [1] if S=G∩F, where G is rg-open and F is rg-closed in (X,τ).

(ii) **rglc^* set** [1] if there exist a rg-open set G and a closed set F of (X,τ) such that S=G∩F.

(iii) **rglc^*^* set** [1] if there exist an open set G and a rg-closed set F of (X,τ) such that S=G∩F.

(iv) **generalized locally closed** (briefly glc) set [3] if S=G∩F, where G is g-open and F is g-closed in (X,τ). The class of all generalized locally closed sets in (X,τ) is denoted by GLC(X,τ).

(v) **GLC^* set** [3] if there exist a g-open set G and a closed set F of (X,τ) such that S=G∩F and

(vi) **GLC^*^* set** [3] if there exist an open set G and a gc-closed set F of (X,τ) such that S=G∩F.

(vii) **g^+-locally closed** [13] (briefly g^+lc) set if S=G∩F, where G is g^+-open in (X,τ) and F is g^+-closed in (X,τ). The class of all g^+-locally closed sets in (X,τ) is denoted by G^+LC(X,τ).

(viii) **G^+LC^* set** [13] if there exists a g^+-open set G and a closed set F of (X,τ) such that S=G∩F and

(ix) **G^+LC^*^* set** [13] if there exists an open set G and a g^+-closed set F of (X,τ) such that S=G∩F.

(x) **g^+-locally closed** [16] (briefly g^+lc) set if S=G∩F, where G is g^+-open in (X,τ) and F is g^+-closed in (X,τ). The class of all g^+-locally closed sets in (X,τ) is denoted by G^+LC(X,τ).

(xi) **G^+LC^* set** [16] if there exists a g^+-open set G and a closed set F of (X,τ) such that S=G∩F.
(xii) **GLC** [16] set if there exists an open set G and a g*-closed set F of (X,τ) such that S=G∩F.
(xiii) g**-locally closed** [15] (briefly g**lc**) set if S=G∩F, where G is g**-open in (X,τ) and F is g**-closed in (X,τ).
The class of all g**-locally closed sets in (X,τ) is denoted by **GLC**(X,τ).
(xiv) **GLC** [15] set if there exists a g**-open set G and a g**-closed set F of (X,τ) such that S=G∩F.
(xv) **GLC** [15] set if there exists an open set G and a g**-closed set F of (X,τ) such that S=G∩F.

**Definition 2.3.** A topological space (X,τ) is called
(i) **submaximal** if every dense subset is open and
(ii) **submaximal** [1] if every dense subset is g**-open.

**Definition 2.4.** A function f:(X,τ)→(Y,σ) is called
(i) **LC-continuous** [4] if f**((V)€GLC(X,τ)) for each open set V of (Y,σ).
(ii) **GLC-continuous** [3] if f**((V)€GLC**‘(X,τ)) for each open set V of (Y,σ).
(iii) **LC-continuous** [13] if f**((V)€GLC**(X,τ)) for each open set V of (Y,σ).
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(xv) **LC-continuous** [16] if f**((V)€GLC**(X,τ)) for each open set V of (Y,σ).
(xvi) **LC-continuous** [16] if f**((V)€GLC**(X,τ)) for each open set V of (Y,σ).

3. g**p-LOCALLY CLOSED SETS**

**Introduction** defined below

**Definition 3.1.** A subset S of a space (X,τ) is called g**-pre locally closed** if S=G∩F, where G is g**p-open and F is g**p-closed in (X,τ).
The class of all g**-pre locally closed sets in (X,τ) is denoted by **G**P**(X,τ).**

**Definition 3.2.** For a subset S of (X,τ), **G**P**(X,τ)** if there exists a g**p-open set G and a closed set F of (X,τ) such that S=G∩F.

**Definition 3.3.** For a subset S of (X,τ), **G**P**(X,τ)** if there exist an open set G and a g**p-open set F of (X,τ) such that S=G∩F.

**Proposition 3.4:**
(i) If S€GLC(X,τ), then S€G**P**(X,τ), S€GLC**(X,τ) and S€G**P**(X,τ) and S€G**P**(X,τ).
(ii) If S€LC(X,τ), then S€G**P**(X,τ), S€GLC**(X,τ) and S€G**P**(X,τ) and S€G**P**(X,τ).
(iii) If S€GLC(X,τ) {[resp. G** LC**‘(X,τ) and G** LC**(X,τ)}], then S€G**P**(X,τ), S€GLC**(X,τ) and S€G**P**(X,τ).
(iv) If S€GLC(X,τ) {[resp. G** LC**‘(X,τ) and G** LC**(X,τ)}], then S€G**P**(X,τ), S€GLC**(X,τ) and S€G**P**(X,τ).
(v) If S€GLC(X,τ) {[resp. G** LC**‘(X,τ) and G** LC**(X,τ)}], then S€G**P**(X,τ), S€GLC**(X,τ) and S€G**P**(X,τ).

vi.i. If S€GLC(X,τ) {[resp. G** LC**‘(X,τ) and G** LC**(X,τ)}], then S€G**P**(X,τ), S€GLC**(X,τ) and S€G**P**(X,τ).

vii. If S€GLC(X,τ) {[resp. G** LC**‘(X,τ) and G** LC**(X,τ)}, then S€G**P**(X,τ), S€GLC**(X,τ) and S€G**P**(X,τ).

viii. If S€GLC(X,τ) {[resp. G** LC**‘(X,τ) and G** LC**(X,τ)}, then S€G**P**(X,τ), S€GLC**(X,τ) and S€G**P**(X,τ).

The proof is obvious from the definitions 2.2, 3.1, 3.2 and 3.3.

The converses of the proposition 3.4 need not be true as seen from the following examples.

**Example 3.5.** Let X=({a,b,c},σ), Y=({a,b},τ), then X**P**(X,Y) and X**P**(Y,X).

**Example 3.6.** Let X=({a,b,c},σ), Y=({a,b},τ), then X**P**(X,Y) and X**P**(Y,X).

**Example 3.7.** Let X and Y be as in the example 3.6, then X**P**(X,Y) and X**P**(Y,X).

**Example 3.8.** Let X=({a,b,c},σ), Y=({a,b},τ), then X**P**(X,Y) and X**P**(Y,X).

**Example 3.9.** Let X=({a,b,c},σ), Y=({a,b},τ), then X**P**(X,Y) and X**P**(Y,X).

**Example 3.10.** Let X and Y be as in the example 3.9, then X**P**(X,Y) and X**P**(Y,X).

**Example 3.11.** Let X and Y be as in the example 3.8, then X**P**(X,Y) and X**P**(Y,X).

**Theorem 3.12.** For a subset S of (X,τ) the following are equivalent
(i) S€G**P**(X,τ).
(ii) S=P∩cl(S) for some g**p-open set P.
(iii) cl(S)-S is g**p-open.
(iv) SU(X-cl(S))=cl(S) for some g**p-open set P.
(v) S€G**P**(X,τ).

**Proof.**
(i) Let S€G**P**(X,τ). Then there exist a g**p-open set P and a closed set F in (X,τ) such that S=P∩F. Since S≤ P and S≤ cl(S), we have S=P∩cl(S). Conversely, since cl(S)≤ F, P∩cl(S)≤ P∩cl(S), we have that S=P∩cl(S).

(ii) => (i) Since P is g**p-open and cl(S) is closed, we have P∩cl(S)€G**P**(X,τ).

(iii) => (iv) Let f€cl(S)-S. By assumption F is g**p-closed. X-F=X∩F=cl(S)-S=SU(X-cl(S)). Since F is g**p-open, we have that SU(X-cl(S)) is g**p-open. Then X-U is g**p-closed. X-U-S=SU(X-cl(S))=S=SU(S). Therefore S=Ucl(S) for the g**p-open set U.

(iv) => (ii) Let U=SU(X-cl(S)). By assumption, U is g**p-open. Now U∩cl(S)=SU(X-cl(S))∩cl(S)∪SU(S)∪SU(S)=SU(S). Therefore S=Ucl(S) for the g**p-open set U.

**Theorem 3.13.** A topological space (X,τ) is called g**p-submaximal** if every dense set is g**p-open.

**Theorem 3.14.** Let (X,τ) be a topological space. Then (i) If (X,τ) is submaximal, then it is g**p-submaximal.
(ii) If \((X, \tau)\) is rg-submaximal, then it is \(g^p\)-submaximal.

**Remark 3.15:** The converses of the theorem 3.14 need not be true as seen from the following examples.

**Example 3.16:** Let \(X = \{a, b, c\}\) and \(\tau = (\Phi, X, \{a, \{a, c\}\})\). \((X, \tau)\) is \(g^p\)-submaximal but not submaximal.

**Example 3.17:** Let \(X\) and \(\tau\) be as in the example 3.9, \((X, \tau)\) is \(g^p\)-submaximal but not \(g^p\)-submaximal.

**Corollary 3.18:** A topological space \((X, \tau)\) is \(g^p\)-submaximal if and only if \(g^p\)-closed subset of \((X, \tau)\) is \(g^p\)-closed.

**Theorem 3.21:** If \(A\in G\) and \(A\) is \(g^p\)-closed, then \(A\) is \(p\)-closed.

**Proof:** Let \(S\in G\) and \(S\subseteq A\). Since \((X, \tau)\) is \(g^p\)-closed, \(S\) is \(g^p\)-closed.

**Theorem 3.22:** If \((X, \tau)\) is \(g^p\)-submaximal, then \(X\) is \(g^p\)-closed.

**Proof:** Let \(A\subseteq X\) be a closed set in \((X, \tau)\). Since \((X, \tau)\) is \(g^p\)-submaximal, \(A\subseteq X\) is \(g^p\)-closed.

**Theorem 3.23:** If \(A\subseteq X\) is \(g^p\)-closed, then \(A\) is \(g^p\)-closed.

**Proof:** Let \(S\in G\) and \(S\subseteq A\). Since \((X, \tau)\) is \(g^p\)-closed, \(S\) is \(g^p\)-closed.

**Theorem 3.24:** If \(A\subseteq X\) is \(g^p\)-closed, then \(A\) is \(g^p\)-closed.

**Proof:** Let \(S\in G\) and \(S\subseteq A\). Since \((X, \tau)\) is \(g^p\)-closed, \(S\) is \(g^p\)-closed.

**Theorem 3.25:** If \(Z\) is closed and open in \((X, \tau)\) and \(A\in G\) and \(A\subseteq Z\), then \(A\) is \(g^p\)-closed.

**Proof:** Let \(A\subseteq Z\). Then there exist a \(g^p\)-open set \(G\) and a \(g^p\)-closed set \(F\) such that \(A\subseteq G\cap F\). Since \((X, \tau)\) is \(g^p\)-closed, \(A\subseteq G\cap F\) is \(g^p\)-closed.

**Theorem 3.26:** If \(Z\) is \(g^p\)-closed, open subset of \((X, \tau)\) and \(A\in G\) and \(A\subseteq Z\), then \(A\) is \(g^p\)-closed.

**Proof:** Let \(A\subseteq Z\). Then there exist a \(g^p\)-open set \(G\) and a \(g^p\)-closed set \(F\) such that \(A\subseteq G\cap F\). Since \((X, \tau)\) is \(g^p\)-closed, \(A\subseteq G\cap F\) is \(g^p\)-closed.

**Proposition 3.27:** If \(A\subseteq X\) and \(B\subseteq X\), then \(A\cap B\) is \(g^p\)-closed.

**Proof:** Let \(S\in G\) and \(S\subseteq A\). Since \((X, \tau)\) is \(g^p\)-closed, \(S\) is \(g^p\)-closed.

**Theorem 3.28:** If \(Z\) is \(g^p\)-closed, then \(Z\subseteq X\).

**Proof:** Let \(S\in G\) and \(S\subseteq Z\). Since \((X, \tau)\) is \(g^p\)-closed, \(S\) is \(g^p\)-closed.

**Theorem 3.29:** If \(A\subseteq X\) is \(g^p\)-closed, then \(A\subseteq X\).

**Proof:** Let \(S\in G\) and \(S\subseteq A\). Since \((X, \tau)\) is \(g^p\)-closed, \(S\) is \(g^p\)-closed.

**Definition 4.1:** A function \(f: (X, \tau)\) is called \(G^p\)-continuous if \(f^{-1}(V)\) is \(G^p\)-closed for every \(V\subseteq X\).

**Definition 4.2:** A function \(f: (X, \tau)\) is called \(G^p\)-irresolute if \(f^{-1}(V)\) is \(G^p\)-open for every \(V\subseteq X\).

**Proposition 4.3:** If \(f\) is \(GLC\)-continuous, then it is \(G^p\)-continuous.

**Theorem 4.4:** If \(f\) is \(G^p\)-continuous, then it is \(G^p\)-irresolute.

**Definition 4.5:** A function \(f: (X, \tau)\) is called \(G^p\)-irresolute if \(f^{-1}(V)\) is \(G^p\)-closed for every \(V\subseteq X\).

**Theorem 4.6:** If \(f\) is \(G^p\)-irresolute, then it is \(G^p\)-continuous.
(ii) If $f$ is LC-continuous, then it is $G^{\text{PLC}}, G^{\text{PLC}^*}$ and $G^{\text{PLC}^*}$.  
(iii) If $f$ is $G^a$ LC [resp. $G^a$ LC* and $G^a$ LC*] -continuous, then it is $G^{\text{PLC}}, G^{\text{PLC}^*}$ and $G^{\text{PLC}^*}$-continuous. 
(iv) If $f$ is $G^a$ LC[resp. $G^a$ LC* and $G^a$ LC*] -continuous, then it is $G^{\text{PLC}}, G^{\text{PLC}^*}$ and $G^{\text{PLC}^*}$-continuous. 
(v) ) If $f$ is $G^a$ LC [resp. $G^a$ LC* and $G^a$ LC*] -continuous, then it is $G^{\text{PLC}}, G^{\text{PLC}^*}$ and $G^{\text{PLC}^*}$-continuous.

**Proof:** Follows from the proposition 3.4, definitions, theorem 17 and theorem 1 of Arockiarani et al. [1]. The converses of the proposition 4.3 need not be true as seen from the following examples.

**Example 4.4:** Let $X=\{a,b,c\}=Y, \tau=\{\Phi, X, \{a,b\}\}$ and $\sigma=\{\Phi, Y, \{b,c\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a)=a, f(b)=b$ and $f(c)=c$. Then it is $G^{\text{PLC}}, G^{\text{PLC}^*}$ and $G^{\text{PLC}^*}$-continuous. But it is not $\text{GLC}^*$-continuous and LC-continuous.

**Example 4.5:** Let $X=\{a,b,c\}=Y, \tau=\{\Phi, X, \{a\}\}$ and $\sigma=\{\Phi, Y, \{a\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a)=a, f(b)=b$ and $f(c)=c$ then it is $G^{\text{PLC}}, G^{\text{PLC}^*}$ and $G^{\text{PLC}^*}$-continuous. But it is not $G^a$ LC, $G^a$ LC and $G^a$ LC* continuous.

**Example 4.6:** Let $X=\{a,b,c\}=Y, \tau=\{\Phi, X, \{a\}\}$ and $\sigma=\{\Phi, Y, \{a\}\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a)=a, f(b)=b$ and $f(c)=c$ then it is $G^{\text{PLC}}, G^{\text{PLC}^*}$ and $G^{\text{PLC}^*}$-continuous. But it is not $G^a$ LC, $G^a$ LC and $G^a$ LC* continuous.

**Theorem 4.8:** Let $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \eta)$ be any two functions. Then

(i) $g\circ f$ is $G^p$ PLC-irresolute if $f$ and $g$ are $G^p$ PLC-irresolute.

(ii) $g\circ f$ is $G^p$ PLC*-irresolute if $f$ and $g$ are $G^p$ PLC*-irresolute.

(iii) $g\circ f$ is $G^p$ PLC***-irresolute if $f$ and $g$ are $G^p$ PLC***-irresolute.

(iv) $g\circ f$ is $G^p$ PLC**-continuous if $f$ is $G^p$ PLC-continuous and $g$ is continuous.

(v) $g\circ f$ is $G^p$ PLC***-continuous if $f$ is $G^p$ PLC*-continuous and $g$ is continuous.

(vi) $g\circ f$ is $G^p$ PLC**-continuous if $f$ is $G^p$ PLC**-continuous and $g$ is continuous.

(vii) $g\circ f$ is $G^p$ PLC*-continuous if $f$ is $G^p$ PLC*-continuous and $g$ is $G^p$ PLC-continuous.

(viii) $g\circ f$ is $G^p$ PLC*-continuous if $f$ is $G^p$ PLC*-continuous and $g$ is $G^p$ PLC**-continuous.

(ix) $g\circ f$ is $G^p$ PLC**-continuous if $f$ is $G^p$ PLC**-continuous and $g$ is $G^p$ PLC**-continuous.

**Remark 4.7:** The following diagram shows the relationships between $g^p$-locally closed sets and some other sets.

Where $A \implies B$ ($A \implies B$) represents $A$ implies $B$ ($A$ does not imply $B$).

**REFERENCES**


