Discussion on Boundary Layer Flows in View of Double Dispersion Influenced by MHD with Viscous Dissipation

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ABSTRACT

Double dispersion and mixed convection effects on heat and mass transfer in a non-Darcy magneto hydrodynamic non-Newtonian fluid over a vertical surface in a porous medium are studied under the constant temperature and concentration. The governing boundary layer equations are solved numerically by the method of fourth order Runge-Kutta method coupled with shooting technique using MATLAB software. The velocity, temperature concentration and heat and mass transfer profiles are presented graphically for various values of the parameters.

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INTRODUCTION

Thermal and solute transport by fluid flowing through a porous matrix is a phenomenon of great interest from both the theory and application point of view. Heat transfer in the case of homogeneous fluid-saturated porous media has been studied with relation to different applications like dynamics of hot underground springs, thermal heat flow through aquifers, flow of moisture through porous industrial materials, and heat exchanges with fluidized beds. Mass transfer in isothermal conditions has been studied with applications to problems of mixing of fresh and salt water in aquifers, miscible displacements in oil reservoirs, spreading of solutes in fluidized beds and crystal washers, salt leaching in soils, etc. Prevention of salt dissolution into the lake waters near the sea shores has become a serious problem of research.

The study of convective flow through porous media has received a great deal of research interest over the last three decades due to its wide and important applications in environmental, geophysical and energy related engineering problems. Prominent applications are the utilization of geothermal energy, the migration of moisture in fibrous insulation, drying of porous solid, food processing, casting and welding in manufacturing processes. Combined heat and mass transfer by free convection in a Darcian fluid-saturated porous medium has been analyzed by Lai [3]. The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa et al. [4].

Mixed convection on bodies embedded in a non-Darcian porous medium have been extensively studied and reported for flow driven by temperature variations only [5-9]. Magneto hydrodynamics (MHD) was originally applied to astrophysical and geophysical problems, where it is still very important, but more recently to the problem of fusion power, where the application is the creation and containment of hot plasmas by electromagnetic forces, since material walls would be destroyed. Astrophysical problems include solar structure, especially in the outer layers, the solar wind bathing the earth and other planets, and interstellar magnetic fields. The effect of pressure stress work and viscous dissipation in some natural convection flows has been analyzed by Joshi and Gebhart [10]. Mamun et al. [11] studied combined effect of conduction and viscous dissipation on MHD free convection flow along a vertical flat plate. Alim et al. [12] studied the combined effect of viscous dissipation and Joule heating on the coupling of conduction & free convection along a vertical flat plate. Rahman et al. [13] investigated the effects of temperature dependent thermal conductivity on MHD free convection flow along a vertical flat plate with heat conduction. Fand and Brucker [14] investigated the effect of viscous dissipation on free convection in porous medium. The effect of viscous dissipation on the Darcian free convection over a non-isothermal body of arbitrary shape embedded in a saturated porous medium has been studied by Nakayama and Pop [15]. Lai and Kulacki [16] discussed the coupled heat and mass transfer by mixed...
convection from a vertical plate in a saturated porous medium. Many problems of Darcian and non-Darcian mixed convection about a vertical plate had been reported, as in Hsu and Cheng [17]. Effect of Double Dispersion on Mixed Convection Heat and Mass Transfer in Non-Darcian Porous Medium has been studied by Murthy [18]. The present chapter deals with various details of boundary layer flow with the presence of MHD and viscous dissipation in the aspect of double dispersion and Mixed convection effects. During the course of study we give the observation over the profiles of velocity, temperature, concentration effects on double dispersion influenced by MHD and viscous dissipation. Results and discussion about a vertical plate had been reported, as in Hsu and Cheng [17].

2. MATHEMATICAL FORMULATION

Consider a two dimensional mixed convection boundary layer flow in a Darcian porous medium along a vertical permeable plate in the presence of a uniform transverse magnetic field. The plate is maintained at constant temperature $T_w$ and constant concentration $C_w$. The temperature and mass concentration of the ambient medium are assumed to be $T_e$ and $C_e$ respectively, the $x$-coordinate is measured along the plate from its leading edge and the $y$-coordinate normal to it. Then the governing equations for the boundary layer flow along with the heat and mass transfer from the wall $y = 0$ into the fluid saturated porous medium $x \geq 0$ and $y \geq 0$ are given by

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

Momentum equation:

\[
\left[1 + \left(\frac{K_{H1}^2}{\mu}\right)\frac{\partial u}{\partial y} + \left(\frac{C}{\nu\alpha}\right)(\partial v)^2\right] \frac{\partial u}{\partial y} = \frac{g_K}{u} \left(\frac{\partial T}{\partial y} + \beta_C \frac{\partial C}{\partial y}\right) \tag{2}
\]

Energy equation:

\[
\frac{\partial T}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left[\alpha \frac{\partial T}{\partial y}\right] + \frac{\nu}{\nu\alpha}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}
\]

Concentration equation:

\[
\frac{\partial C}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial C}{\partial y} = \frac{\partial}{\partial y}\left(D_C \frac{\partial C}{\partial y}\right) \tag{4}
\]

where $u$ and $v$ are velocities in $x$ and $y$ directions, $T$ is the temperature, $K$ is the permeability constant, $C$ is an empirical constant, $D$ is the kinematic viscosity, $g$ is the acceleration due to gravity, $\beta_C$ is the coefficient of thermal expansion, $\beta_T$ is the coefficient of solute expansion, $M$ is the magnetic field. In equation (2) the “+” sign corresponds to the case of aiding buoyancy and the “–” sign corresponds to the case of opposing buoyancy flow, $\alpha$ is the thermal diffusivity and $D_C$ is the mass diffusivity.

The boundary conditions of the problem are:

\[
y = 0 \quad ; u = 0, T = T_e, C = C_e \quad \text{near the plate} \\
y \to \infty; u \to u_e, T = T_e, C = C_e \quad \text{far from the plate} \tag{5}
\]

Now using the dimensionless quantities

\[
\psi = f(x)(\alpha_{xw} x)^{\frac{1}{2}}, q = \frac{F(x)}{\sqrt{x}}, \theta = \frac{T - T_e}{T_e - T_v}, \phi = \frac{C - C_e}{C_v - C_e}
\]

where

\[
\psi \quad \text{is the stream function where} \quad u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}
\]

the governing equations (2) - (4) together with the boundary conditions (5) are transformed as

\[
(1 + M + F f') f' = \pm \frac{R_m}{P_e} \left(\theta + N \phi\right) \tag{6}
\]

\[
\phi' + \frac{1}{2} Le f \theta + Le B \left(f' \phi' + f \phi\right) = 0 \tag{7}
\]

with the boundary conditions

\[
\eta \to 0 \quad : \quad f = 0, \quad \theta = 1, \quad \phi = 1 \tag{9}
\]

\[
\eta \to \infty \quad : \quad f' \to 1, \quad \theta \to 0, \quad \phi \to 0
\]

where $Ra_m = \frac{Kg \beta_T (T_e - T_v) \eta}{\nu \alpha}$ is the Local Rayleigh number, $M = \frac{Kg \mu^2 H_2^2}{\mu}$ is the magnetic parameter, $Le = \frac{\alpha}{b}$ is the Lewis number, $N = \frac{\beta_C (C_e - C_v)}{\beta_T (T_v - T_e)}$ is the buoyancy ratio, $B = \frac{\xi d u}{\alpha}$ is solute dispersion, $D = \frac{\gamma d u}{\alpha}$ is thermal dispersion, $E_c = \frac{u_c^2}{C_p (T_e - T_v)}$ is the viscous parameter, and $Pr = \frac{\nu}{\alpha}$ is the Prandtl number. Also the local Nusselt number $Nu_e$ and the local Sherwood number $Sh_e$, with which one can characterize the surface heat transfer rate and surface mass transfer rate are defined as follows

\[
Nu_e = -\frac{x}{(T_e - T_v) \frac{\partial T}{\partial y} \bigl|_{y=0}} \tag{10}
\]

\[
Sh_e = -\frac{x}{(C_e - C_v) \frac{\partial C}{\partial y} \bigl|_{y=0}} \tag{11}
\]

RESULTS AND DISCUSSION

The dimensionless equations (6), (7) and (8) together with the boundary conditions (9) are solved numerically by means of the fourth order Runge-Kutta method coupled with the shooting technique with $M$, $Le$, $B$, $F$, $N$, $Ec$, $D$, $(Ra_m/P_e)$ and $Pr$ as prescribed parameters. The choice of $\eta_{max} = 4$ helps to compare the present results with those of the earlier works. The computations were done by a program skill through MATLAB. To get a clear insight of
the physical problem, the velocity, temperature and concentration have been discussed in fig.1 to fig.7 by assigning numerical values to the parameters encountered in the problem. Also the changes in the profiles of heat and mass transfer are traced in fig.8 & fig.9 respectively.

**In case of aiding flow:**

From fig.1 it is observed that in the absence of double dispersion, the velocity decreases rapidly with the increase of the magnetic parameter. But in the case of dispersion the velocity decreases slowly with the increase of the magnetic parameter. Also for a fixed value of the magnetic parameter it is observed that there is more velocity drop in absence of double dispersion. It is found from fig. 2(a) that in the absence of double dispersion, increase in magnetic parameter produces a significant increase in the temperature profile. But such observation is not seen in presence of dispersion. From fig. 3(a) it is noted that both in presence and absence of double dispersion, the concentration increases with increase in magnetic parameter. In fig. 4 one can observe that increase in viscous dissipation, without dispersion, we notice a very slight separation indicating the increase in velocity. In the presence of dispersion the increase in the viscosity parameter shows the corresponding increase in the velocity almost all with the velocities as super imposed.

From fig. 5(a) it is observed that the change in temperature is directly related to the change in the viscosity in the absence of double dispersion. Fig. 6 depicts that in absence of dispersion velocity profile of the Darcian state rapidly decreases up to some extent and then coincides with the corresponding profile of the non-Darcian. But in presence of dispersion, one can notice a uniform variation in both Darcy and non-Darcy velocity profiles. It is found from fig.7 that in absence of double dispersion both the Darcian and non-Darcian patterns of temperature profiles show a uniform decrease. Fig. 8 shows that in presence and absence of dispersion there is a tremendous down fall in heat transfer at the increased value of the convection parameter than at its earlier value as indicated by the heat transfer profiles of the graph. Fig. 9 gives us that in the absence of dispersion mass transfer rate decreases gradually with the decrease in the mixed convection parameter. In the presence of dispersion with the increase of mixed convection parameter the mass transfer rates almost all decrease.

**In case of opposing flow:**

From fig.1 it is noticed that an increase in the magnetic parameter enhances the velocity of the fluid both in presence and absence of double dispersion, but this change is considerable in absence of dispersion. In fig.2 (b), it can be seen that both in presence and absence of the dispersion, the temperature is decreasing with the increase in the value of the magnetic parameter. In fig. 3(b) it is noticed that in absence of dispersion concentration increases with the increase in the magnetic parameter. Taking dispersion into account with the increase in magnetic parameter much uniformity cannot be perceived in concentration variation. Fig. 4 depicts that in presence/absence of double dispersion an increase in viscosity parameter shows a corresponding equal increase in velocity.

From fig. 5(b) it is observed that under dispersion, the variation in the viscosity parameter causes the variation in temperature in a directly proportional way.

From fig. 6 it is noticed that in the absence of double dispersion the slope goes up rapidly indicating sudden increase in velocity profile. Whereas in presence of double dispersion the velocity profiles of both Darcian and non-Darcian situations gradually increase. Fig.7 it can be seen that in absence of double dispersion the Darcian and non-Darcian profile patterns of temperature maintain their uniform decrease throughout. In presence of dispersion the variation in the Darcian and non-Darcian temperature profiles is significant. Fig. 8 it is observed that without dispersion, the heat transfer rate shows a rapid sudden increase in its behavior. With dispersion, when the heat transfer at a higher value is considered even though initially it showed an increase in heat transfer it could not exceed the earlier heat transfer rate taken. Fig. 9 it is noted that the rate of increase in mass transfer is more in absence of dispersion.

![Fig.1 Variation of velocity profile for different values of double dispersion and Magnetic parameter in aiding and opposing flows](image1)

\[ F = 2, N = 1, Le = 1, E_c = 0.6, Pr = 0.73 \]

![Fig.2 (a) Effect of temperature profile for different values of melting and double dispersion on similarity variable in aiding flow](image2)

\[ F = 2, N = 1, Le = 1, E_c = 0.6, \frac{Ra}{Pe} = 1 \]
Fig. 2(b) Effect of temperature profile for different values of melting and double dispersion on similarity variable in opposing flow

\[
F = 2, N = 1, Le = 1, E_c = 0.6, Pr = 0.73, \frac{Ra}{Pe} = -1
\]

Fig. 3(a) Effect of concentration profile for different values of melting and double dispersion on similarity variable in aiding flow

\[
F = 2, N = 1, Le = 1, E_c = 0.6, Pr = 0.73, \frac{Ra}{Pe} = 1
\]

Fig. 3(b) Effect of concentration profile for different values of melting and double dispersion on similarity variable in opposing flow

\[
F = 2, N = 1, Le = 1, E_c = 0.6, Pr = 0.73, \frac{Ra}{Pe} = 1
\]

Fig. 4 Variation of velocity profile for different values of viscous dissipation and double dispersion on similarity variable in aiding and opposing flow

\[
M = 1, F = 2, N = 1, Le = 1, Pr = 0.73
\]

Fig. 5(a) Variation of temperature profile for different values of viscous dissipation and double dispersion on similarity variable in aiding flow

\[
M = 1, F = 2, N = 1, Le = 1, Pr = 0.73, \frac{Ra}{Pe} = 1
\]

Fig. 5(b) Variation of temperature profile for different values of viscous dissipation and double dispersion on similarity variable in opposing flow

\[
M = 1, F = 2, N = 1, Le = 1, Pr = 0.73, \frac{Ra}{Pe} = -1
\]
Sastry et al. Discussion On Boundary Layer Flows In View Of Double Dispersion Influenced By MHD With Viscous Dissipation

Fig. 6 Variation of velocity profile for different values of non-Darcy parameter and double dispersion on similarity variable in aiding and opposing flows
\( (M = 1, N = 1, Le = 1, E_c = 0.6, Pr = 0.73) \)

Fig. 7 Variation of temperature for different values of non-Darcy parameter and double dispersion on similarity variable in aiding and opposing flows
\( (M = 1, N = 1, Le = 1, E_c = 0.6, Pr = 0.73) \)

Fig. 8 Effect of Magnetic parameter on Nusselt number for different values of double dispersion and mixed convection parameter
\( (M = 1, N = 1, Le = 1, E_c = 0.6, Pr = 0.73) \)

Fig. 9 Effect of Magnetic parameter on Sherwood number for different values of double dispersion and mixed convection parameter
\( (M = 1, N = 1, Le = 1, E_c = 0.6, Pr = 0.73) \)

REFERENCES


