The present paper deals with the determination of temperature change, displacement and thermal stresses in a thin annular disc defined by $a \leq r \leq b$ with internal heat generation. Heat dissipates by convection from inner circular surface ($r = a$) and outer circular surface ($r = b$). Also, initially the annular disc is at arbitrary temperature $f(r)$. Here we modify Kulkarni et al. (2008) for homogeneous heat convection along radial direction. The governing heat conduction equation has been solved by the method of integral transform technique. The results are obtained in a series form in terms of Bessel's functions. The results for temperature change, displacement and stresses have been computed numerically and illustrated graphically.
and \( v \) and \( \alpha \) are respectively, the Poisson’s ratio and the linear coefficient of thermal expansion of the thin annular disc material. Introducing \( U_i = \phi_i \) \( i = 1, 2 \)
we have
\[
\nabla_i^2 \phi = (1 + v) \alpha_i T \quad (3)
\]
\[
\nabla_i^2 = \frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y_i^2}
\]
\[
\sigma_{ij} = 2 \mu (\delta_{ij} - \delta_{ij} \phi_{kk}) \quad i, j = 1, 2 \quad (4)
\]
where \( \mu \) is the Lame constant and \( \delta_{ij} \) is the Kronecker symbol.

In the axially-symmetric case
\[ \phi = \phi(r, t), \quad T = T(r, t) \]
and the differential equation governing the displacement potential function \( \phi(r, t) \) is given by
\[
\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial t^2} = (1 + v) \alpha_i T \quad (5)
\]
\[
\text{with the boundary conditions,}
\]
\[
h_{11} T + \frac{\partial}{\partial r} T = 0 \quad \text{at} \quad r = a \quad (6)
\]
\[
h_{22} T + \frac{\partial}{\partial t} T = 0 \quad \text{at} \quad r = b \quad (7)
\]
The stress function \( \sigma_{rr} \) and \( \sigma_{\theta \theta} \) are given by
\[
\sigma_{rr} = -2 \mu \frac{\partial \phi}{\partial r} \quad (8)
\]
\[
\sigma_{\theta \theta} = -2 \mu \frac{\partial \phi}{\partial r} \quad (9)
\]
In the plane state of stress within the annular disc
\[
\sigma_{rr} = \sigma_{zz} = \sigma_{\theta \theta} = 0 \quad (10)
\]
The temperature of the annular disc satisfies the heat conduction equation,
\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{1}{k} \frac{\partial T}{\partial t} \quad (11)
\]
with the boundary conditions,
\[
h_{11} T + \frac{\partial}{\partial r} T = 0 \quad \text{at} \quad r = a, t > 0 \quad (12)
\]
\[
h_{22} T + \frac{\partial}{\partial t} T = 0 \quad \text{at} \quad r = b, t > 0 \quad (13)
\]
and the initial condition
\[
T = f(r) \quad \text{at} \quad t = 0, a \leq r \leq b \quad (14)
\]
where \( k \) is the thermal conductivity of the material of the disc, \( \alpha \) is the thermal diffusivity of the material of the disc, \( q(r, t) \) is internal heat generation and \( h_{11} \) and \( h_{22} \) are the relative heat transfer coefficients on the inner and outer surface of the thin annular disc.

Equations (1) to (14) constitute the mathematical formulation of the problem.

**SOLUTION OF THE HEAT CONDUCTION EQUATION**

To obtain the expression for temperature \( T(r, t) \), we introduce the finite Hankel transform over the variable \( r \)
and its inverse transform defined in Ozisik (1968) are
\[
\hat{T}(\beta_m, r) = \int_{r=a}^{b} r K_0(\beta_m r) T(r, t) dr \quad (15)
\]
\[
T(r, t) = \sum_{m=1}^{\infty} K_0(\beta_m r) \hat{T}(\beta_m, r) \quad (16)
\]
where \( K_0(\beta_m r) = \frac{J_0(\beta_m r)}{\sqrt{\beta_m}} \quad (17) \)

and
\[
R_0(\beta_m r) = \left[ \frac{J_0(\beta_m r)}{\beta_m J_0(\beta_m a) + h_{22} \beta_m J_0(\beta_m b)} - \frac{Y_0(\beta_m r)}{\beta_m Y_0(\beta_m a) + h_{22} \beta_m Y_0(\beta_m b)} \right] \quad (18)
\]

The normality constant
\[
N = \frac{b^2}{2} \left( 1 + \frac{h_{22}^2}{\beta_m} \right) R_0^2(\beta_m a) - \frac{a^2}{2} \left( 1 + \frac{h_{22}^2}{\beta_m} \right) R_0^2(\beta_m b) \quad (19)
\]
and \( \beta_1, \beta_2, \beta_3 \ldots \) are the positive roots of the transcendental equation
\[
\frac{\beta_m J_0(\beta_m a) + h_{22} \beta_m J_0(\beta_m b)}{\beta_m J_0(\beta_m a) + h_{22} \beta_m J_0(\beta_m b)} - \frac{\beta_m Y_0(\beta_m a) + h_{22} \beta_m Y_0(\beta_m b)}{\beta_m Y_0(\beta_m a) + h_{22} \beta_m Y_0(\beta_m b)} = 0 \quad (20)
\]
where \( J_0(\chi) \) is Bessel function of the first kind of order \( n \) and \( Y_0(\chi) \) is Bessel function of the second kind of order \( n \).

On applying the finite Hankel transform defined in the Eq. (16) and its inverse transform defined in Eq. (17), one obtains the expression for temperature as
\[
T(r, t) = \sum_{m=1}^{\infty} e^{-\alpha \beta_m^2 t} K_0(\beta_m r) \int_{r=a}^{b} r K_0(\beta_m r) q(r, t) dr \quad (21)
\]

**DISPLACEMENT POTENTIAL AND THERMAL STRESSES**

Using Eq. (22) in Eq. (5), one obtains
\[
\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} = (1 + v) \alpha_i \sum_{m=1}^{\infty} e^{-\alpha \beta_m^2 t} K_0(\beta_m r) \int_{r=a}^{b} r K_0(\beta_m r) q(r, t) dr \quad (22)
\]

Solving Eq. (23), one obtains
\[
\phi(r, t) = -\left( 1 + v \right) \alpha_i \sum_{m=1}^{\infty} \frac{1}{\beta_m} e^{-\alpha \beta_m^2 t} K_0(\beta_m r) \int_{r=a}^{b} r K_0(\beta_m r) q(r, t) dr \quad (23)
\]

Using Eq. (24) in Eqs. (8) and (9), one obtains expression for thermal stresses as
\[
\sigma_{rr} = 2 \mu \left( 1 + \beta \right) \alpha_i \sum_{m=1}^{\infty} \frac{1}{\beta_m} e^{-\alpha \beta_m^2 t} \int_{r=a}^{b} r K_0(\beta_m r) f(r) dr \quad (24)
\]

\[
\sigma_{\theta \theta} = -2 \mu \left( 1 + \beta \right) \alpha_i \sum_{m=1}^{\infty} \frac{1}{\beta_m} e^{-\alpha \beta_m^2 t} \int_{r=a}^{b} r K_0(\beta_m r) q(r, t) dr \quad (25)
\]

**SPECIAL CASE AND NUMERICAL CALCULATIONS**

Setting
\[
F(\beta_m) = \frac{1}{\sqrt{N}} \left( b J_1(\beta_m b) - a J_1(\beta_m a) - 2 b J_2(\beta_m b) - a^2 J_2(\beta_m a) \right) \quad (2)
\]

\[
q = q_i \delta(r - r_0) \delta(t - \tau) \quad (2)
\]
where \( r \) is the radius measured in meter, \( \delta(r) \) is well known dirac delta function of argument \( r \).

The internal heat source \( q(r, t) \) is an instantaneous line heat source of strength \( q_i \) situated at the centre of the thin annular disc along the radial direction and releases its heat instantaneously at the time \( \tau = 10 \) sec.

Material Properties

The numerical calculation has been carried out for a copper (pure) annular disc with the material properties defined as,

Thermal diffusivity \( \alpha = 112.34 \times 10^{-6} \text{m}^2 \text{s}^{-1} \),
Specific heat $c_p = 383 J/Kg$, 
Thermal conductivity $k = 386 W/m K$
Shear modulus $G = 48 Gpa$,  
Youngs modulus $E = 130 Gpa$,  
Poisson ratio $\nu = 0.35$,  
Coefficient of linear thermal expansion $a_t = 16.5 \times 10^{-6}m^2/s^1$,  
Lamé constant $\mu = 26.67$

**Roots of Transcendental Equation**
The $\beta_1 = 3.8214, \beta_2 = 7.0232, \beta_3 = 10.1672, \beta_4 = 13.3292, \beta_5 = 16.4657$ are the roots of transcendental equation

$$\frac{\beta_m \partial^2 \phi}{\partial \theta^2} + \frac{\beta_m \partial \phi}{\partial \theta} + \frac{\beta_m \partial^2 \phi}{\partial \phi^2} = 0.$$  
The numerical calculation and the graph has been carried out with the help of mathematical software Matlab.

**DISCUSSION**

In this paper a thin annular disc is considered and determined the expressions for temperature, displacement and stresses due to internal heat generation within it. As a special case mathematical model is constructed for considering copper (pure) thin annular with the material properties specified above.

From figure 1, it is observed that temperature increases for $1 \leq r \leq 1.2, 1.4 \leq r \leq 1.6$ and $1.8 \leq r \leq 2$ along radial direction. Temperature decreases for $1.2 \leq r \leq 1.4$ and $1.6 \leq r \leq 1.8$. Overall behavior of temperature along radial direction is decreasing from the inner circular surface to outer circular surface of a thin annular disc.

From figure 2, it is observed that the displacement function $\phi$ decreases for $1 \leq r \leq 1.2, 1.4 \leq r \leq 1.6$ and $1.8 \leq r \leq 2$ along radial direction. It increases for $1.2 \leq r \leq 1.4$ and $1.6 \leq r \leq 1.8$ along radial direction. Overall behavior of displacement function along radial direction is decreasing from the inner circular surface to outer circular surface of a thin annular disc.

From figure 3, it is observed that the radial stress $\sigma_{rr}$ decreases from for $1 \leq r \leq 1.2, 1.4 \leq r \leq 1.6$ and $1.8 \leq r \leq 2$ along radial direction. It increases for $1.2 \leq r \leq 1.4$ and $1.6 \leq r \leq 1.8$ along radial direction. Overall behavior of radial stress $\sigma_{rr}$ is tensile towards outer circular surface along radial direction.

From figure 4, it is observed that the angular stress function $\sigma_{\theta \theta}$ for $1 \leq r \leq 1.2, 1.4 \leq r \leq 1.6$ and $1.8 \leq r \leq 2$ along radial direction. It increases for $1.2 \leq r \leq 1.4$ and $1.6 \leq r \leq 1.8$ along radial direction. Overall behavior of angular stress function $\sigma_{\theta \theta}$ is compressive towards outer circular surface along radial direction.

**CONCLUSION**

We can conclude that due to instantaneous internal heat generation, temperature and displacement decreases from the inner circular surface to outer circular surface along radial direction of thin annular disc, whereas the radial stress $\sigma_{rr}$ is tensile in nature and the angular stress function $\sigma_{\theta \theta}$ is compressive in nature along radial direction of thin annular disc. The results obtained here are useful in engineering problems particularly in the determination of state of stress in a thin annular disc, base of furnace of a thermal power plant, gas power plant and the measurement of aerodynamic heating.

**REFERENCES**


