NUMERICAL STUDY OF LIQUID METAL MHD FLOW THROUGH A SQUARE DUCT

Dipjyoti Sarma, P. N. Deka, S. C. Kakaty

Department of Mathematics, Dibrugarh University, Dibrugarh-786004, Assam
Department of Statistics, Dibrugarh University, Dibrugarh-786004, Assam

ABSTRACT

A numerical study has been carried out for steady MHD flow of liquid metal through a square duct under the action of strong transverse magnetic field. The pair of walls parallel to the magnetic field is considered to be electrically conducting while other pair of walls is insulating. The numerical solutions for velocity and induced magnetic field have been obtained by using 5 point stencil central difference scheme. Distribution of velocity and induced magnetic field are studied from 3D graphs obtained for different values of Hartmann number \((M)\). The velocity and induced magnetic field contours in the duct flows are presented for several values of \(M\).

INTRODUCTION

The study of liquid metal flows through ducts under the influence of strong transverse magnetic field is gaining its importance due to its industrial application such as magnetic pump, electromagnetic flow meter, magnetic pump, casting of steel, conduction pump etc. In designing liquid metal tritium breeder coolant blanket for nuclear fusion reactor adequate knowledge of basic understanding of liquid metal MHD is necessary, particularly, analysis on flow associated with very high Hartmann number is important. The full set of equations governing liquid metal MHD including very thin Hartmann boundary layer equations are very difficult to solve analytically. Therefore the development of efficient numerical technique to study various issues related to such a duct flow are required.

During last few decades, numerical investigations on the duct flow of liquid metal under action of magnetic field was carried out by several researchers. Kim et al. (1989) developed a numerical procedure based on finite difference method to obtain the full solution of liquid metal MHD duct flow relevant to fusion blankets. Liquid metal flow through conducting circular duct under action of a transverse magnetic field that varies in the flow direction was studied using a computationally efficient and fast numerical method by McCarthy et al. (1989).

Kim and Abdou (1989) developed a numerical algorithm by using finite volume method to present the characteristics of fluid flow and heat transfer in a rectangular duct with constant wall temperature for relatively large values of Hartmann number and interaction parameter. Study of two dimensional as well as three dimensional liquid metal MHD flow through rectangular duct under action of inclined strong magnetic field was carried out by means of numerical simulation technique by Sterl (1990). Ying and Tillack (1991) made an analysis on the issue of heat transfer for flow of liquid metal through elongated rectangular ducts under fusion environment by using finite element method. In relatively complex geometries, for the analysis of liquid metal MHD duct flow Kunugi et al. (1991) developed a new computer code KAT having the potentiality to solve many MHD fluid flow and heat transfer problems. McCarthy and Abdou (1991) developed an iterative technique based numerical method for analyzing the flow of liquid metal MHD flow in multiple adjacent ducts. Tezer-Sezgin (1994) employed boundary element method to study the flow of incompressible, viscous and electrically conducting fluid in a rectangular duct with conducting walls parallel to applied magnetic field and his computation was carried out for Hartmann number up to 10. For the MHD channel flow with arbitrary conducting walls, boundary element method was used by Tezer-Sezgin and Dost (1994) for several values of Hartmann number \((1 \leq M \leq 10)\) and conductivity parameter \((0 \leq \lambda \leq \infty)\). Hua and Walker (1995) made numerical investigations on the flow of liquid metal though rectangular under the action of non uniform transverse magnetic field and hence obtained numerical solution for fringing magnetic field. For the study of MHD channel flow at moderate Hartmann numbers Demendy and Nagy (1997) proposed a new analytic finite element method for solving the governing equations. Barrett (2001) made an analysis on the flow of conducting fluid through a straight channel with variable conductivity walls under the action of uniform magnetic field by finite element method.

For the analysis of MHD pressure drop issue associated with the flow of liquid metal through channels having sandwich structure of several materials Smolentsev et al. (2005) developed a numerical code based on finite volume formulation. Bhuyan and Goswami (2008) investigated the effect of magnetic field on MHD pressure drop that arises in the flow of liquid metal through a
rectangular duct by finite volume method. Tezer-Sezgin
and Bozakaya (2008) obtained BEM solution of
magnetohydrodynamic flow in a rectangular duct with one
conducting walls parallel to applied magnetic field with the
consideration of values of Hartmann number up to 300.
Recently, Albets-Chico et al. (2011) studied the flow of
liquid metal in a poorly conducting pipe subjected to a
strong fringing method by high resolution numerical
simulation based on finite volume method.

In this paper numerical investigations have been
made for liquid metal MHD flow through a square duct
under the action of strong transverse magnetic field which is
acting normal to the walls. The walls to which transverse
field acts normally are considered to be insulating walls
and other walls as conducting walls. The velocity and
induced magnetic field distribution associated with this
liquid metal MHD duct flow environment are analysed by
using a 5-point stencil finite central difference scheme.

**Formulation of Problem**

In this problem we are considering the steady
motion of incompressible liquid metal through a square
duct under the action of strong transverse magnetic field
applied perpendicular to two opposite walls of the duct.
The pair of walls parallel to the magnetic field is conducting
while other pair of wall is insulating.

**Fig 1: Geometrical model of the flow problem**

Following assumptions are made:
I. The flow is fully developed,
II. Hartmann walls are treated as insulating walls and
III. Side walls are assumed to be conducting.
Under these assumptions, the velocity, magnetic field and
temperature will be of the form

\[ \vec{V} = [0,0,V_z(x,y)] \]
\[ \vec{B} = [0,B_y,B_z(x,y)] \]

The governing equations of the flow are

**Equation of Continuity:**

\[ \nabla \cdot \vec{V} = 0 \] (1)

**Momentum Equation:**

\[ \rho \left( \frac{\partial \vec{V}}{\partial t} + \nabla \cdot (\vec{V} \times \vec{B}) \right) + \nabla P = \vec{J} \times \vec{B} + \mu N^2 \vec{V} \] (2)

**Ohms law:**

\[ \vec{J} = \sigma \left( \vec{E} + \nabla \times \vec{B} \right) \] (3)

**Amperes law**

\[ \nabla \times \vec{B} = \mu_e \vec{J} \] (4)

**Faradays law** (for steady flow)

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = 0 \] (5)

Using Eq.(4) in Eq.(2) for this velocity distribution and
assuming

\[ \frac{\partial P}{\partial z} = -P \text{(constant)} \]

We finally obtain

\[ \rho N^2 V_i \frac{\partial B_i}{\partial y} = -\mu \] (6)

Using Eq. (3) and Eq. (5), Eq. (4) reduces to

\[ \lambda \nabla^2 B_z + B_0 \frac{\partial V_z}{\partial y} = 0 \] (7)

The non dimensional quantities used in this problem are
defined as follows:

\[ x' = \frac{x}{L}, B' = \frac{B}{B_0}, y' = \frac{y}{L}, R_m = \mu_i \sigma V_0 L \]

The non-dimensional equations are

\[ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + M^2 \frac{\partial B}{\partial y} = -1 \] (9)

\[ \frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2} + R_m \frac{\partial V}{\partial y} = 0 \] (10)

The boundary conditions considered are:

\[ x = \pm 1: V = 0; \frac{\partial B}{\partial x} = 0 \]
\[ y = \pm 1: V = 0; B = 0 \] (11)

Here it is observed that Eq. (9) and Eq. (10) are coupled
non-linear equations which are to be solved using
boundary condition given in Eq. (11). In view of
complexities in seeking closed form solutions, numerical
solutions are considered. These equations are expressed in
finite difference equation by utilizing a 5-point stencil
centered finite difference scheme with mesh size \( h = 1/N \)
where \( N \) is a pre-assigned positive integer. The resulting
finite difference representation for Eq. (9) and Eq. (10) are
as follows:

\[ V_{i,j} = 0.25 \left[ V_{i+1,j} + V_{i-1,j} + V_{i,j+1} + V_{i,j-1} \right] + C_1 B_{i,j-1} - B_{i,j+1} + C_2 \]

\[ B_{i,j} = 0.25 \left[ B_{i+1,j} + B_{i-1,j} + B_{i,j+1} + B_{i,j-1} \right] + C_3 \left( V_{i,j+1} - V_{i,j-1} \right) \] (12)

**Where**

\[ C_1 = \frac{h M^2}{8 R_m}, C_2 = \frac{h^2}{4}, C_3 = \frac{h R_m}{8} \]

The numerical boundary conditions for present duct flow
are as follows:

For the case of insulating walls \( y = \pm 1 \)

\[ V(i,1) = 0, V(i,N+1) = 0, B(i,1) = 0, B(i,N+1) = 0; \] (14a)

and for conducting walls \( x = \pm 1 \)

\[ V(1,j) = 0, V(N+1,j) = 0, B(1,j) = B(3,j); \]

\[ B(N+1,j) = B(N-1,j); \] (14b)

The convergence of each of the computed value of variable
$V, B$ at different grid points is checked by using root-meansquare residuals $R_i$ (Sun(2007)) for each flow variable.

RESULTS AND DISCUSSION

The velocity and induced magnetic field profiles for liquid metal MHD duct flow inside a square duct are obtained by the numerical solutions of Eq. (12) and Eq. (13) using boundary condition given in Eq.(14) for finite difference scheme with M=50, 100, 200 and 300.

The contour plots for induced magnetic field are shown in Fig.'s 2, 3, 4 and 5 for M=50,100,200 and 300 respectively. From the contour plotting it is observed that there is a formation of boundary layers near the insulated walls. This observation is same as obtained by Tezer-Sezgin and Bozkaya (2008) using BEM method. Again from the contour plots for velocity given in Fig.'s 6,7,8 and 9 for M=50,100,200 and 300, it is observed that with the increasing values of Hartmann number $M$ there is faster boundary layer formation for velocity near the insulating walls as compared to the conducting walls, which also tally with the earlier results obtained by Tezer-Sezgin and Bozkaya (2008) using BEM method.

Again from the contour plots for induced magnetic field given in Fig.'s 2,3,4 and 5 for M=50,100,200 and 300, it is observed that there is formation of boundary layers near the insulated walls which also tally with the earlier results obtained by Tezer-Sezgin and Bozkaya (2008) using BEM method.
In the duct, with the increase of magnetic field strength the phenomena of pressure drop due to increased $\mathbf{J}\times\mathbf{B}$ force become intense. From the 3D plotting of velocity given in Figs. 10, 11 and 12 for $M=100, 200$ and 300 this pressure drop phenomena is clearly visible with the increasing values of Hartmann numbers.

The numerical technique used in present study is efficient and reliable and capable of dealing with high Hartmann flow. The computed results of this investigation are in good agreement with BEM solution of MHD duct flow obtained by Tezer-Sezgin and Bozkaya (2008).

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