UNBALANCED FUZZY TRANSPORTATION PROBLEM WITH ROUBAST RANKING TECHNIQUE

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ABSTRACT

The aim of Fuzzy transportation is to find the least transportation cost of some commodities through a capacitated network when the supply and demand of nodes and the capacity and cost of edges are represented as fuzzy numbers. Many previous papers 6,7,12,13 have presented arithmetic operations, alpha level and simple ranking by operations. P. Pandian11 has presented some methods for fuzzy transportation problem. Finally all these research papers present the solutions of balanced FTP by alpha level, simple ranking methods and simple arithmetic operations. In this paper we proposed a ranking method to find the fuzzy optimal solution of unbalanced fuzzy transportation problems occurring in real life situations. Also, a new representation of trapezoidal fuzzy numbers is proposed. The advantages of the proposed methods over existing methods and the proposed representation of trapezoidal fuzzy numbers over existing representation are also discussed.

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INTRODUCTION

In today’s highly competitive market, the pressure on organizations to find better ways to create and deliver value to customers becomes stronger. Howand when to send the products to the customers in the quantities, they want in a cost-effective manner, become more challenging. Transportation models provide a powerful framework to meet this challenge. They ensure the efficient movement and timely availability of raw materials and finished goods. The basic transportation problem was originally developed by Hitchcock 4. In conventional transportation problems it is assumed that decision maker is aware of the precise values of transportation cost, availability and demand of the product. In real world applications, all the parameters of the transportation problems may not be known precisely due to uncontrollable factors. This type of imprecise data is not always well represented by random variable selected from probability distribution. Fuzzy numbers introduced by Zadeh17 may represent this data. So, fuzzy decision making method is needed here. Zimmermann18 showed that solutions obtained by fuzzy linear programming are always efficient. Subsequently, Zimmermann’s fuzzy linear programming has developed into several fuzzy optimization methods for solving transportation problems. Oheigartaigh* proposed an algorithm for solving transportation problems where the capacities and requirements are fuzzy sets with linear or triangular membership functions. Chanaset al14 presented a fuzzy linear programming model for solving transportation problems with crisp cost coefficients and fuzzy supply and demand values. Chanas and Kuchta4 proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers, and developed an algorithm for obtaining the optimal solution. Saad and Abbas discussed the solution algorithm for solving the transportation problem in fuzzy environment. Liu and Kao7 described a method for solving fuzzy transportation problems based on extension principle. Gani and Razak3 presented a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy numbers. Dinagar and Palanivel3 investigated fuzzy transportation problem, with the aid of trapezoidal fuzzy numbers and proposed fuzzy modified distribution method to find the optimal solution in terms of fuzzy numbers. Pandian and Natarajan11 proposed a new algorithm namely, fuzzy zero point method for finding a fuzzy optimal solution for a fuzzy transportation problem, where the transportation cost, supply and demand are represented by trapezoidal fuzzy numbers.

PRELIMINARIES

In this section some basic definitions and arithmetic operations are reviewed.
Basic definitions

1. **Fuzzy set**: A fuzzy set is characterized by a membership function mapping element of a domain, space or universe of discourse X to the unit interval [0, 1] i.e. \( A = \{ (x, \mu_A(x); x \in X) \}. \) Here \( \mu_A : X \rightarrow [0,1] \) is a mapping called the degree of membership function of the fuzzy set \( A \) and \( \mu_A(x) \) is called the membership value of \( x \) in the fuzzy set \( A \). These membership grades are often represented by real numbers ranging from [0,1].

2. **Trapezoidal fuzzy numbers**
   - A fuzzy number \( \tilde{A} = (m, n, a, b) \) is said to be trapezoidal fuzzy number if its membership function is given by
     \[
     \mu(\tilde{A})(x) = \begin{cases} 
     0 & x < m \\
     (x-m)/(n-m) & m \leq x \leq n \\
     (b-x)/(b-a) & a \leq x \leq b \\
     0 & x > b 
     \end{cases}
     \]
   - If \( m=0, \, n=0, \, a=0, \, b=0 \), then \( \tilde{A} \) is a zero fuzzy number.
   - If \( m=0, \, n=0, \, a=0, \, b=1 \), then \( \tilde{A} \) is a unit fuzzy number.
   - If \( m=n=0, \, a=0, \, b=1 \), then \( \tilde{A} \) is a triangular fuzzy number.

3. **Properties of Trapezoidal fuzzy number**
   - 1. **Trapezoidal fuzzy nu. \( \tilde{A} = (m,n,a,b) \) is said to be non negative trapezoidal fuzzy nu. Iff \( m,a \geq 0 \)
   - 2. A trapezoidal fuzzy nu. \( \tilde{A} = (m,n,a,b) \) is said to be zero trapezoidal fuzzy nu. Iff \( m=0, n=0, a=0, b=0 \).
   - 3. Two trapezoidal fuzzy numbers \( \tilde{A}_1 = (m_1,n_1,a_1,b_1) \) and \( \tilde{A}_2 = (m_2,n_2,a_2,b_2) \) are said be equal I.e. \( \tilde{A}_1 = \tilde{A}_2 \) Iff \( m_1=m_2, n_1=n_2, a_1=a_2, b_1=b_2 \).

4. **Arithmetic Operators for solving Trapezoidal fuzzy number**
   - If \( A = (m_1, n_1, a_1, b_1) \) and \( B = (m_2, n_2, a_2, b_2) \) two trapezoidal fuzzy numbers then the arithmetic operations on \( A \) and \( B \) as follows:
     - Addition \( A+B = (m_1+m_2, n_1+n_2, a_1+a_2, b_1+b_2) \)
     - Subtraction \( A-B = (m_1-b_2, n_1-a_2, a_1-n_2, b_1-m_2) \)
     - Multiplication \( A*B = (t_1, t_2, t_3, t_4) \)
       - Where \( t_1 = \min \{ m_1 m_2, m_1 b_2, m_2 b_1, b_1 b_2 \} \)
       - \( t_2 = \min \{ n_1 n_2, n_1 a_2, a_1 n_2, a_1 a_2 \} \)
       - \( t_3 = \max \{ n_1 n_2, n_1 a_2, a_1 n_2, a_1 a_2 \} \)
       - \( t_4 = \max \{ m_1 m_2, m_1 b_2, m_2 b_1, b_1 b_2 \} \)

5. **Robust Ranking Technique**: Roubast ranking technique which satisfy compensation, linearity, and additivity properties and provides results which are consistent with human intuition. If \( \tilde{a} \) is a fuzzy number then the Roubast Ranking is defined by

\[
R(\tilde{a}) = \int_0^1 \left( a_{\alpha L} \right)^{\alpha} \left( a_{\alpha U} \right)^{1-\alpha} \, d\alpha
\]

where \( a_{\alpha L} \) and \( a_{\alpha U} \) are the \( \alpha \) level cut of the fuzzy number \( \tilde{a} \) and \( \alpha \) is in [0,1].

In this paper we use this method for ranking the subjective values. The Roubast ranking index \( R(\tilde{a}) \) gives the representative value of fuzzy number \( \tilde{a} \).

**FUZZY TRANSPORTATION PROBLEM**

In conventional transportation problems it is assumed that decision maker is sure about the precise values of transportation cost, availability and demand of the product. In real world applications, all these parameters of the transportation problems may not be known precisely due to uncontrollable factors. For example, in real life problems the following situation may occur:

(a) Let a product is to be transported first time at a destination and no expert have knowledge about the transportation cost then there exist uncertainty about the transportation cost.

(b) If a new product is launched in the market then there always exists uncertainty about the demand of that particular product.

(c) In daily life problems, it can be seen that whenever a customer ask to a supplier that the particular product is available or not, sometimes supplier answers yes it is available but after a few seconds supplier answers sorry at this time this product is not available. Sometimes a supplier does not have any uncertainty about the statement that the product is available or not. When a customer demands for a particular product the supplier answer yes, the product is available, but if the demand of product is large then again supplier says, I check that so much quantity is available or not i.e. their exist uncertainty about the availability of product. To deal with such situations, fuzzy set theory is applied in literature to solve the transportation problems.

Several authors (Liu and Kao 2004; Dinagar and Palanivel 2009; Pandian and Natrajan 2010) have proposed different methods for solving balanced fuzzy transportation problems by representing the transportation cost, availability and demand as normal fuzzy numbers. The balanced fuzzy transportation problem, in which a decision maker is uncertain about the precise values of transportation cost, availability and demand, may be formulated as follows:

\[
\text{Minimize } \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}
\]

Subject to \( \sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, 3, ..., p \)

\( \sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, 3, ..., q \)

\( \sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j \)

\( x_{ij} \) is a non-negative trapezoidal fuzzy number

Where \( p \) is total number of sources

\( Q \) is total number of destinations

\( a_i \) is the fuzzy availability of the product at ith source

\( b_j \) is the fuzzy demand of the product at jth destination

\( c_{ij} \) is the fuzzy transportation cost for unit quantity of the product from ith source to jth destination

\( x_{ij} \) is the fuzzy quantity of the product that should be transported from ith source to jth destination.

To minimize the total fuzzy transportation cost:

\( \sum_{i=1}^{p} a_i = \) total fuzzy availability of the product

\( \sum_{j=1}^{q} b_j = \) total fuzzy demand of the product

\( \sum_{i=1}^{p} a_i = \sum_{j=1}^{q} b_j \) then the fuzzy transportation problem is said to be balanced fuzzy transportation problem, otherwise it is called unbalanced fuzzy transportation problem.

**Numerical Example**

A company has four sources \( S_1, S_2, S_3 \) and \( S_4 \) and four destinations \( D_1, D_2, D_3 \) and \( D_4 \). The fuzzy transportation cost for unit quantity of the product from \( i^\text{th} \) source to \( j^\text{th} \) destination is \( c_{ij} \) where

\[
c_{ij} = \begin{cases} 
4.6,7,9 & (3.5,7,10) \\
4.8,9 & (3.5,7,10) \\
4.8,7 & (3.5,7,10) \\
4.6,9 & (3.5,7,10) 
\end{cases}
\]

where \( \{c_{ij}\}_{i,j=4} \) is shown in the following table:

<table>
<thead>
<tr>
<th>Source</th>
<th>Destination</th>
</tr>
</thead>
<tbody>
<tr>
<td>S_1</td>
<td>D_1</td>
</tr>
<tr>
<td></td>
<td>D_2</td>
</tr>
<tr>
<td></td>
<td>D_3</td>
</tr>
<tr>
<td></td>
<td>D_4</td>
</tr>
</tbody>
</table>

And fuzzy availability of the product at source are (4.5,7,10), (3.5,7,9), (7,10,13), (5.7,10,14)) and the fuzzy demand of the product at destinations are ((4,10,13,15), (3.7,9,13), (2.3,10,14), (3.7,10,13)) respectively.

The fuzzy transportation problems are...
The above fuzzy transportation problem is an unbalanced fuzzy transportation problem. Then convert the unbalanced problem into balanced problem as follows:

**Solution:** In Conformation to model the fuzzy transportation problem can be formulated in the following mathematical programming form

\[
\text{Min } Z = R(4,6,7,9)x_{11} + R(3,5,7,10)x_{12} + R(5,7,10,12)x_{12} + R(5,7,9,13)x_{13} + R(3,4,6,9)x_{14} + R(2,3,5,9)x_{21} + R(5,7,9,13)x_{22} + R(4,6,9,12)x_{23} + R(5,6,7,10)x_{34} + R(7,9,10,12)x_{31} + R(6,7,9,10)x_{32} + R(7,9,10,13)x_{33} + R(6,7,10,13)x_{34} + R(4,5,7,9)x_{41} + R(5,7,12,15)x_{42} + R(7,9,13,15)x_{43} + R(2,4,10,13)x_{44}.
\]

\[R(\bar{a}) = \int_{0}^{1} (0.5)a^{a}_{x} a^{a}_{y} da\]

\[R(4,6,7,9) = \int_{0}^{1} (0.5)(2a + 4.9 - 2a) da\]

\[R(4,6,7,9) = \int_{0}^{1} (0.5)(2a + 4 + 9 - 2a) da\]

Similarly

\[R(4,6,7,9) = 6.5 \times R(3,5,7,10) = 6.25, R(5,7,10,12) = 8.5, R(3,4,6,9) = 5.75\]

\[R(2,3,5,9) = 4.75, R(5,7,9,13) = 8.5, R(4,6,9,12) = 7.75, R(5,6,7,10) = 7\]

\[R(7,9,10,12) = 9.5, R(6,7,9,10) = 8, R(7,9,10,13) = 9.75, R(6,7,10,13) = 9\]

\[R(4,5,7,9) = 6.25, R(5,7,12,15) = 9.75, R(7,9,13,15) = 11, R(2,4,10,13) = 7.25\]

**Rank of all supply**

\[R(4,5,7,10) = 6.5, R(2,3,5,7) = 4.25, R(7,10,13,14) = 11, R(5,7,10,14) = 9, R(7,15,6,10) = 9.5\]

**Rank of all Demands**

\[R(4,10,13,14) = 10.25, R(3,7,9,13) = 8, R(2,3,10,14) = 7.25, R(3,7,10,13) = 8.25, R(13,13,0,0) = 6.5\]

**REFERENCES**

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