SKOLEM GRACEFUL SIGNED GRAPHS ON DIRECTED GRAPHS

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**ABSTRACT**

In this paper, a new concept skolem graceful signed graphs on directed graphs has been introduced. A graph \( G(p, m, n) \) is a bijective function \( f: V(G) \rightarrow \{1,2,3,\ldots,p\} \) such that when each edge \( uv \in E(G) \) is assigned by \( f(uv) = f^+(v) - f^-(u) \) the positive edges receive distinct labels from the set \( \{1,2,3,\ldots,m\} \) and the negative edges receive distinct labels from the set \( \{-1,-2,-3,\ldots,-n\} \) is called a skolem graceful signed graphs. Moreover, some families of graphs which has skolem signed graphs are being coming under investigation.

**INTRODUCTION**

All Graphs in this paper are finite and directed. The symbols \( V(G) \) and \( E(G) \) denote the vertex set and edge set of a graph \( G \). The cardinality of the vertex set is called the order of \( G \) denoted by \( p \). The cardinality of the edge set is called the size of \( G \) denoted by \( q \) edges is called a \((p,q)\) graph. A graph labeling is an assignment of integers to the vertices or edges. Bloom and Hsu[2] extended the notion of graceful labeling to directed graphs. Graceful signed graphs \( f(uv) \) is the difference between \( f^+(v) \) and \( f^-(v) \)

\[ f(uv) = |f^+(v) - f^-(u)| \]

where \( f^+(v) \) is the sum of the labels of all arcs with \( v \) as head and \( f^-(v) \) is the sum of the labels of all arcs with \( u \) as tail. In this paper, edges receives positive and negative signs.

**Definition 1.1**

Let \( G \) be a simple graph with order \( p \) and size \( q \). A function \( f: V(G) \rightarrow \{1,2,3,\ldots,p\} \) is called a graceful labeling if

i) \( f \) is one-to-one.

ii) the edges receive all the labels from \( 1 \) to \( q \), where the label of an edge is computed to be the absolute value of the difference between the vertex labels at its ends i.e., if \( e = (u,v) \) then the label of \( e \) is \( |f(u) - f(v)| \).

**Definition 1.2**

If \( f: V(G) \rightarrow \{1,2,3,\ldots,p\} \) is a bijective mapping and \( f(uv) = |f(u) - f(v)| \) for all \( uv \in E(G) \). If \( f^+(E) = \{1,2,3,\ldots,q\} \), then \( f \) is called skolem graceful labeling.

**Definition 1.3**

A signed graph is a graph where edges are assigned positive or negative sign.

**Definition 1.4**

A graph \( G(p,m,n) \) is a bijective function \( f: V(G) \rightarrow \{1,2,3,\ldots,p\} \) such that when each edge \( uv \in E(G) \) is assigned by \( f(uv) = f^+(v) - f^-(u) \) the positive edges receive distinct labels from the set \( \{1,2,3,\ldots,m\} \) and the negative edge receive distinct labels from the set \( \{-1,-2,-3,\ldots,-n\} \) is called a skolem graceful signed graphs.

**Theorem 1.1**

For any positive integer \( n \), a path \( p_n \) is skolem graceful signed graphs.

**Proof**

The path consists of \( n \) vertices and \( n-1 \) edges. Let \( V(G) \) denotes the set of all vertices of \( G \) i.e., \( V(G) = \{v_1,v_2,v_3,\ldots,v_n\} \) Let \( E(G) \) denotes the set of all edges of \( G \) i.e., \( E(G) = \{e_1,e_2,e_3,\ldots,e_n\} \) Define \( f: V(G) \rightarrow \{1,2,3,4,\ldots,p\} \) as follows

\[ f(v_i) = \begin{cases} i+1 & \text{if } i \text{ is odd} \\ (n+1)-i & \text{if } i \text{ is even} \end{cases} \]

\[ f(e_i) = \begin{cases} i & \text{if } i \text{ is odd} \\ -i & \text{if } i \text{ is even} \end{cases} \]

The path \( p_n \) receives equal number of positive and negative edges when \( n \) is odd.

Therefore, the path \( p_n \) is skolem graceful directed signed graphs.

**Theorem 1.2**

For every positive integer \( n \), the star graph \( K_{1,n} \) is skolem graceful signed graphs.

**Proof**

The star graph \( G \) consists of \( n+1 \) vertices and \( n \) edges. Let \( v_0 \) be the centre vertex. If \( v_0 \) be the smallest value among \( v_1,v_2,v_3,\ldots \) it receives positive edges whereas \( v_0 \) is the largest value among \( v_1,v_2,v_3,\ldots \) it receives negative edges.

Define \( f: V(G) \rightarrow \{1,2,3,4,\ldots,p\} \) as follows

\[ f(v_0) = 1 \]

\[ f(v_i) = i+1; 1 \leq i \leq n \]

\[ f(e_i) = i; 1 \leq i \leq n \]

Let \( f(v_0) = n \)
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G₁ receives positive edges and G₂ receives negative edges. Therefore, the star graph $k_{1,n}$ is a skolem graceful signed graphs.

**Theorem 1.3**
For every positive integer $n$, the flower graph is skolem graceful signed graphs.

**Proof**
The flower graph consists of $n+2$ vertices. Let $V(G)$ denotes the set of all vertices and $E(G)$ denotes the set of all edges. Define $f: V(G) \rightarrow \{1,2,3,\ldots,p\}$ as follows:

$$ f(v_i) = \begin{cases} i+1 & \text{if } i \text{ is odd} \\ \frac{(n+1) - i}{2} & \text{if } i \text{ is even} \end{cases} $$

Therefore, a flower graph is skolem graceful signed graphs.

**Theorem 1.4**
S graph is skolem graceful signed graphs off $n$ is even.

**Proof**
S graph consists of $n$ vertices and $n-1$ edges. Let $V(G)$ denotes the set of all vertices and $E(G)$ denotes the set of all edges. Define $f: V(G) \rightarrow \{1,2,3,\ldots,p\}$ as follows:

$$ f(v_i) = \begin{cases} \frac{i+1}{2} & \text{if } i \text{ is odd} \\ \frac{(n+1) - i}{2} & \text{if } i \text{ is even} \end{cases} $$

Therefore, S graph is skolem graceful signed graphs.

**CONCLUSION**
In this paper, a formula for labelings in skolem signed directed graphs has been established. Further, it has been proved that certain families of graphs are skolem signed directed graphs.

**REFERENCES**