INTRODUCTION

Image segmentation plays a vital role in Vision and Image processing applications. It is used widely in areas such as document image analysis, scene or map processing. Satellite imaging and material inspection in quality control tasks are examples of applications that employ image thresholding or segmentation to extract useful information from images. Medical image processing is another area that has extensively used image thresholding to help the experts to better interpret digital images for a more accurate diagnosis or to plan treatment.

Segmentation based on gray level histogram thresholding is a method to divide an image containing two regions if interest; object and background. In fact, applying this threshold to the whole image, pixels whose gray level is under this value are assigned to a region and the remainder to the other. When the histogram doesn’t exhibit a clear separation, ordinary thresholding techniques might perform poorly. Fuzzy set theory provides a new tool to deal with multimodal histograms.

The rest of the paper is organized as follows. Section 2 discusses about well-known segmentation methods in the literature. Section 3 explains the general definitions followed by construction of GLSC Histogram in Section 4. Section 5 explains proposed methodology and Section 6 demonstrates the results and final conclusions.

THRESHOLDING ALGORITHMS

The process of segmentation separates the object from background of the image. All similar pixels belongs to the object are separated from the rest of the image, 2D Gray level histogram is used to segment the image. Many popular techniques are used in this category. In ideal cases the histogram shows a deep valley between two peaks, each represents either an object or background and the threshold falls in the valley region as in fig. 1. But some images will not express clear separation of the pixels as two peaks, where threshold computation is a difficult task. To address this problem several methods have been proposed in literature [1][2]. Otsu [3] proposed discriminant analysis to maximize the separability of the resultant classes. An iterative selection method is proposed in reference [4]. J.Kittler and J.Illingworth's[5] proposed minimum error Thresholding method. Entropy based algorithms proposed by Kapur et al.[6] propose a method based on the previous work of pun[7] that first applied the concept of entropy to Thresholding. His methods concludes when the sum of the background and object entropies reaches its maximum, the image threshold is obtained. In Kapur et al. Images which are corrupted with noise or irregular illumination produce multimodal histograms in which a 2D histogram does not guarantee the optimum threshold selection process, because no spatial correlation is considered. Entropy criterion function is applied on 3D GLSC histogram to optimize threshold by surpassing difficulties with 2D histogram [8,9]. This work is further enhanced by Seetharama Prasad et al.[10] with variable similarity measure producing improved GLSC Histogram. In reference [11] Type-2 fuzzy is used with GLSC histogram with human visual nonlinearity characteristics to identify the optimal similarity measure. The ordinary Thresholding techniques perform poorly where, non-uniform illumination corrupts object characteristics and inherent Image vagueness is present. Fuzzy based Image Thresholding methods are introduced in literature to
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overcome this problem. Fuzzy set theory[12] is used in these methods to handle grayness ambiguity and vagueness during the process of threshold selection. Several segmentation algorithms based on fuzzy sets are found in the literature [12]-[16]. Several segmentation algorithms based on fuzzy sets are found in the literature based on Fuzzy measure, which is a measure of a vagueness in the image used in many segmentation algorithm[17], ultrafuzziness a new measure introduced in [18] as type-II fuzzy, the gray level intensity value is selected to be the optimum threshold at which the fuzzy index is minimized. An unsupervised segmentation method is proposed without any maximization criteria using fuzzy measures in [19]. Haung and wang [20] assigns a membership degree to each pixel in the image, and the image is considered as a fuzzy set and the membership distribution explains each pixel belongs to either objet set or background set in the misclassification region of the histogram. Seetharama Prasad et al. [21] proposed a new method over N.V.Lopes et al. of [19] which suffers for low contrast images. A sample test image and its histogram is shown in fig.1

Fig 1: Cameraman image and its histogram

GENERAL DEFINITIONS

1. Fuzzy S-function

\[ S(x; a, b, c) = \begin{cases} 0 & x < a \\ 2 \left( \frac{x-a}{c-a} \right)^2 & a \leq x \leq b \\ 1 - 2 \left( \frac{x-c}{c-a} \right)^2 & b \leq x \leq c \\ 1 & x \geq c \end{cases} \] (1)

For object pixels, \( \mu_O(x) = S(x; a, b, c) \)

\[ \mu_O(x) = \begin{cases} 0 & x < a \\ 2 \left( \frac{x-a}{c-a} \right)^2 & a \leq x \leq b \\ 1 - 2 \left( \frac{x-c}{c-a} \right)^2 & b \leq x \leq c \\ 1 & x \geq c \end{cases} \] (2)

For background pixels, \( \mu_B(x) = S(x; a, b, c) \)

\[ \mu_B(x) = \begin{cases} 1 & x < a \\ 1 - 2 \left( \frac{x-a}{c-a} \right)^2 & a \leq x \leq b \\ 2 \left( \frac{x-a}{c-a} \right)^2 & b \leq x \leq c \\ 0 & x \geq c \end{cases} \]

Also \( \mu_B (x) = 1 - \mu_O (x) \) is true.

Fig 2: Shape of the S-function

Where \( b = (a + c) / 2 \) parameters a and c controls S-function shown in fig.3. The three parameters a, b and c satisfies the condition \( 0 \leq a \leq b \leq c \leq 255 \).

2 Initial Fuzzy seed subset

From reference [21] initial fuzzy seed subset values a, b and c are estimated. Let \( x(i,j) \) be the gray level intensity of image at \((i,j)\). \( I = \{ x(i,j) | i \in [1,Q], j \in [1,R] \} \) is an image of size Q x R, i.e. N. The gray level set \{0,1,2,...,255\}. The mean(\( \mu \)) and standard deviation(\( \sigma \)) are calculated as follows

\[ \mu = \frac{1}{N} \sum_{i=1}^{N} x(i,j) \times h(i) \] (3)

\[ \sigma = \frac{1}{N} \sqrt{ \sum_{i=1}^{N} (x(i,j) - \mu)^2 } \] (4)

From Equations (3) and (4) fuzzy seed set values a, b and c are estimated as

\[ b = \mu \] (5)

\[ a = \mu - \sigma \] (6)

\[ c = \mu + \sigma \] (7)

3 Type-II fuzzy

Uncertainty In image thresholding based on fuzzy type I concern to the assignment of a membership degree to a pixel. Membership functions as the kernel of fuzzy sets are defined by the expert and are based on his knowledge. One can distinguish different fuzzy techniques through their membership function definitions. To consider the effect of all these functions and find a robust solution, type II fuzzy sets have been introduced and are applicable. In order to extend our type I fuzzy entropic thresholding to type II form, we require defining a type II fuzzy set. For this propose we need a type I fuzzy set and construct the footprint of uncertainty by assigning upper and lower membership degrees to each point (Fig. 4).

In type II fuzzy set \( X \), for all \( x \in X \) we create a three dimensional membership function that characterizes a type-II fuzzy set. This is called the footprint of uncertainty. Therefore, fuzzy sets of type II have fuzzy type I membership function instead of crisp membership functions [12]. A type II fuzzy set, \( \hat{A} \), is characterized by a type II membership function \( \hat{\mu} \). A more practical definition for a type II fuzzy set can be given as follows:

Here we do as Type II Interval Fuzzy Sets and set all secondary grades equal 1, so we have:

\[ \hat{A} = \{(x, \mu_L(x), \mu_U(x)) | \exists x \in X, \mu_L(x) \leq \mu(x) \leq \mu_U(x), \mu \in [0, 1]\} \]

Using dilation and concentration, the upper and lower membership degrees \( \mu_L \) and \( \mu_U \) from fig.4 of initial membership function \( \mu \) can be calculated as:

Fig 3: Multimodal image histogram and the characteristic functions for the seed subsets.
\[ \mu_0(x) = \mu(x)^{0.5}, \quad (8) \]
\[ \mu_L(x) = \mu(x)^2 \quad (9) \]

Now we require a type reducer to obtain proper membership degree. In this case, type II interval fuzzy sets, we can use centroid type reducer. Here the centroid is the midpoint of its domain and as we have two value \( \mu_0 \) and \( \mu_L \), it will be:
\[ \mu = (\mu_0 + \mu_L)/2 \quad (10) \]

Fig. 4 A possible way to construct type II fuzzy sets. The interval between lower/left and upper/right membership values (bounded region) will capture the footprint of uncertainty.

4 Membership degrees as Fuzzy probabilities

Let \( D=\{(i,j)\}: i=0,1,\ldots,m-1; j=0,1,\ldots,n-1 \} \) and \( G=\{0,1,\ldots,L-1\} \), where \( m, n \) and \( L \) are three positive integers, for gray scale images \( L = 256 \). \( D \) can be segmented into \( D_0 \) as the object, pixels with high gray values and \( D_B \) as the background, pixels with low gray values from fig. 5. \( \Phi = \{D_0, D_B\} \) has unknown probabilistic partition \( P_0 \) and \( P_B \).

Fig. 5: Fuzzy membership functions graph

Therefore \( P_0 = \sum_{x=0}^{t} P_x * P_{0|x} \quad (11) \)
\[ P_B = \sum_{x=t+1}^{255} P_x * P_{B|x} \quad (12) \]

Where \( P_{0|x} \) and \( P_{B|x} \) are the conditional probabilities of the object and background respectively.

The type-II fuzzy membership functions \( \mu_O(x) \) and \( \mu_B(x) \)(x) can be used to approximate the conditional probabilities / fuzzy probabilities [15], therefore equations (11) and (12) can be written as:
\[ P_0 = \sum_{x=0}^{t} P_x * \mu_O(x) \quad (13) \]
\[ P_B = \sum_{x=t+1}^{255} P_x * \mu_B(x) \quad (14) \]

Zadeh[12] suggests a definition about the entropy of a fuzzy set which takes both distribution and the membership into consideration and with the introduction of type-II fuzzy sets, is defined as follows:

\[ H(A) = \sum_{i=0}^{255} \mu_A(x_i) \log(\mu_A(x_i) \mu_B(x_i)) \]

where \( A \) is a fuzzy set; \( \mu_A(x) \) is the type-II membership function of element \( x \), which has a probability distribution of \( p(x) \).

GLSC Histogram

Spatial correlation histogram differentiates two images in which the frequencies of intensities are exactly same, but they differ in their placements, where two dimensional histograms fail. Therefore different images with identical two-dimensional histogram will result in the same threshold value. To overcome the problem GLSC histogram is constructed by considering the similarity in neighborhood pixels with some adaptive threshold value as similarity measure (\( \zeta \)) as always by Yang Xiao et al. [8][9] this is further improved by Seetharama Prasad M et al.[10].

1 Novel similarity measure (\( \zeta \))

References[8][9] considered only local properties of the image and stated similarity measure \( \zeta \) as constant at 4 producing reasonable results. Reference[10] have employed global properties into account along with local properties in computation of similarity measure \( \zeta \) as, for every \( NXN \) map, with the help of Otsu’s discrimination analysis to measure image global and local characteristic \( C_g \) and \( C_l \) respectively.

\[ \zeta = 2*|C_g - C_l| \]

Due to the computational penalty of the Otsu method, we have adopted a statistical parameter Standard Deviation to decide the similarity measure for every \( NXN \) map, keeping global standard deviation unchanged. Therefore \( \zeta \) is computed as the difference between global standard deviation of the entire image and standard deviation of local \( NXN \) map.

\[ \zeta = |\text{std}_g - \text{std}_l| \]

Fig 6: GLSC Histogram (with constant \( \zeta=4 \)) of Cameraman image

2 Computation of GLSC histogram

Let \( f(x,y) \) be the gray level intensity of image at \( (x,y) \). \( F = \{f(x,y)|x \in [1,Q], y \in [1,R]\} \) of size \( Q \times R \). The gray level set \( \{0,1,2,\ldots,255\} \) is considered as \( G \) throughout this paper for convenience. The image GLSC histogram is computed by taking only image local properties into account as follows. Let \( g(x, y) \) be the similarity count corresponding to pixel of image \( f(x, y) \) in \( N \times N \) neighborhood, where \( N \) is any positive odd number in range \( [3, \min(Q/2,R/2)] \).

\[ g(x+1,y+1) = \]

6
where, \( \mathcal{O} \) is the empty set and \( \mathcal{O} \neq \mathcal{O} \).

GLSC histogram is constructed as shown in fig.7 with the correlated probability at different gray level intensities from equations (15) and (16) as follows.

\[
\begin{align*}
\text{GLSC histogram} &= \{ (f(x, y), p(f(x, y))) \mid x, y \in \mathcal{M} \} \\
&= \{ (f(x, y), P(f(x, y))) \mid x, y \in \mathcal{M} \}
\end{align*}
\]

where, \( P(f(x, y)) = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x, y) \)

(17)

Where, \( P \) is the gray level correlation probability computed for all pixels with intensity \( k \in \mathcal{G} \) with correlation \( m \in \{1, 2, \ldots, N \} \) and histogram is normalized.

3 Non linear Weight function.

**Fig 8: Weight plot**

Where, **Weight** \((m, N)\) is a non linear function as shown in fig. 8 applied to GLSC histogram elements for entropic calculation derived as.

\[
\text{Weight}(m, N) = \frac{1 + \frac{e^{-m}}{m}}{1 + \frac{e^{-m}}{m}}
\]

(18)

Where, \( N \) is any positive odd number in range \([3, \min(Q/2, R/2)]\) and \( m \in \{1, 2, \ldots, N \} \), Fig. 8 shows **Weight** \((m, N)\) a plot when \( N=3 \). Probability function \( p(k, m) \) is calculated as:

\[
p(k, m) = \frac{\text{No. of pixels with gray value } k \text{ and } m \text{ correlation}}{\text{total No. of pixels in image}}
\]

(19)

Where, \( \sum_{k=0}^{255} \sum_{m=1}^{N} p(k, m) = 1 \)

(20)

4 Probability computations

The two disjoint sets corresponding to object and background are \( \mathcal{G}_o, \mathcal{G}_b \) are partitioned from set \( \mathcal{G} \) corresponding to entire image with \( t \) the probability distribution associated with object and background are given by

\[
\begin{align*}
\hat{p}(l,j) &= \{ p(1, j) / (255^N) \} \forall l \in \{0, 255\}, j \in \{1, N\} \} \\
&= \left\{ \begin{array}{c}
\hat{p}(0,1) \quad \hat{p}(0,255) \\
\hat{p}(1,1) \quad \hat{p}(1,255) \\
\hat{p}(255,1) \quad \hat{p}(255,255) \\
\end{array} \right\}
\end{align*}
\]

(21)

where, \( \hat{p}(l,j) = \sum_{k=0}^{255} \sum_{m=1}^{N} \hat{p}(k, m) \)

(22)

Weight \((m, N)\) is computed by Equation (8).

\[
\text{Weight}(m, N) = \left\{ \begin{array}{c}
\text{Object} \\
\text{Background} \\
\end{array} \right\}
\]

(23)

where, \( \text{Weight}(m, N) = \sum_{k=0}^{255} \sum_{m=1}^{N} p(k, m) \)

(24)

and \( \text{P}_o(t) = 1 - \text{P}_b(t) \)

(25)

5 Entropy maximization

Kapur et al. [6] criterion function is used to estimate the threshold on GLSC histogram instead of 2D gray level histogram with a defined weight for the taken map \( N=3 \).

\[
\phi(t, N) = H_0(t, n) + H_b(t, n)
\]

(26)

where, \( \phi(t, N) \) is the entropic criterion function, \( H_0(t, n) \) and \( H_b(t, n) \) are entropies associated with object and background respectively and optimal threshold \( t^* \) can be obtained by maximizing \( \phi(t) \).

\[
t^* = \arg \max \{ \phi(t) \}
\]

(27)

From equation (5) \( H_0, H_b \) are entropies associated with object and background distribution.

\[
H_0(t, N) = \sum_{k=0}^{N-N} \sum_{m=1}^{N} p(k, m) \ln \left( \frac{p(k, m)}{p_b(t)} \right) \ast \text{Weight}(m, N)
\]

(28)

And

\[
H_b(t, N) = \sum_{k=0}^{N-N} \sum_{m=1}^{N} p(k, m) \ln \left( \frac{p(k, m)}{p_b(t)} \right) \ast \text{Weight}(m, N)
\]

(29)

Fig. 1 shows GLSC of ‘cameraman.tif’ and Fig. 7 an Improved GLSC Histograms. and fig. 6 is an GLSC histogram with similarity measure as 4. In 2D histogram the two peaks represents object and background, in GLSC histogram the same are represented in 3D, which gives the information about spatial correlation. But it fails to identify the ambiguous region, which is well defined in improved GLSC histogram and it offers profound choice to maximize entropy based criterion function.

**PROPOSED METHOD**

The main problem with fuzzy sets type I, regardless of which shape we use and what algorithm is applied, is that the assignment of a membership degree to an element/pixel is not certain. Membership functions are usually defined by the expert and are based on his intuition/knowledge. The fact that different fuzzy techniques differ mainly in the way that they define the intuition/knowledge. The fact that different fuzzy techniques differ mainly in the way that they define the membership function is for the most part due to this dilemma. To find a more robust solution, type II fuzzy sets should be introduced.

1 Methodology

Now Equations (15) and (16) can be rewritten as

\[
\text{P}_o(t) = \sum_{k=0}^{255} \sum_{m=1}^{N-N} p(k, m) \ast \mu_o(k)
\]

(30)

and

\[
\text{P}_b(t) = 1 - \text{P}_o(t)
\]

(31)

Equations (6) and (7) are now changed to

As with Entropy maximization

\[
H_0(t, N) = \sum_{k=0}^{N-N} \sum_{m=1}^{N} p(k, m) \ast \mu_o(k) \ln \left( \frac{p(k, m)}{p_o(t)} \right) \ast \text{Weight}(m, N)
\]

(32)

and

\[
H_b(t, N) = \sum_{k=0}^{N-N} \sum_{m=1}^{N} p(k, m) \ast \mu_b(k) \ln \left( \frac{p(k, m)}{p_b(t)} \right) \ast \text{Weight}(m, N)
\]

(33)

and background respectively and optimal threshold \( t^* \) can be obtained by maximizing \( \phi(t) \).
RESULTS AND CONCLUSIONS

To illustrate the proposed methodology we consider 14 standard test images as an image set having similar and dissimilar gray level histogram characteristics, varying from uni-modal to multimodal as shown in Fig. 9 and are to compare with gold standard ground truth images which are manually generated. From this group of images skull, Blocks, potatoes, trees, rice, lena, coins, wheel, house, pepper are performed well. The low contrast image emblem did not do good, needs some preprocessing steps. Images like anshu, cameraman and rose yielded better performance but still there is scope to improve further.

As the future scope the proposed method to be tested with some standard methods like Otsu, Kapur entropy and other popular techniques to decide its performance. This fuzzy-II method can also be tested against counter technique fuzzy-I method. Finally it is proved to be one of the good thresholding techniques to segment any gray image.

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M. Seetharama Prasad, did his B.E(cse) from University of Mysore and M.E(cse) from Vinayaka Missions University. He has published eight research papers in various international journals, presently working as Professor in LBRCE, Mylavaram. He has got 15 years of academic experience and he is pursuing his Ph.D in Digital Image Processing.

Dr. R. Satya Prasad
Received Ph.D. degree in Computer Science in the faculty of Engineering in 2007 from A N University, India. His current research is focused on Software Engg., Image Processing & Database Management System. He has published several papers in National & International journals.

CH.Venakata Narayana, did his M.Tech(CSE) from JNT University, Hyd. He is pursuing his Ph.D in DIP from AN University, Guntur. He has published several research papers in various international journals. He is Presently working as an Associate Professor in LBRCE, Mylavaram.